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**Combination of Cold and Hot Components in the Energy Spectra  
of Electrons Scattered by Relativistically Intense Laser Pulses  
with Various Transverse Distributions of Amplitude**

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## Scattering of a Sparse Ensemble of Electrons: Experiment Overview

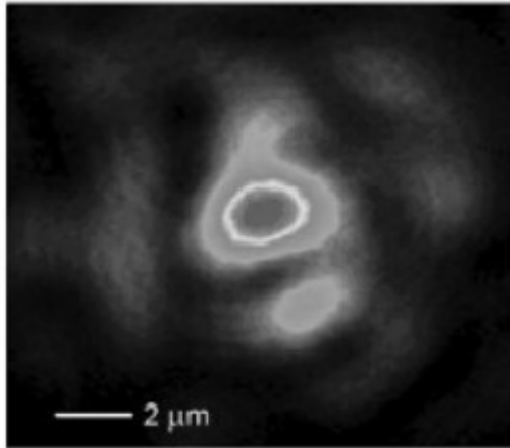
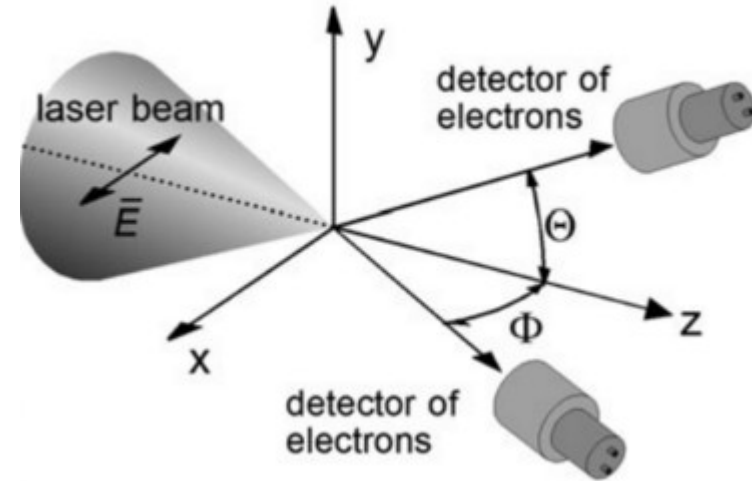


Photo of Laser Intensity Distribution at beam Waist



Schematic of the Experimental Setup

### Max Born Institute for Short-Time Spectroscopy, Berlin, Germany

#### Laser:

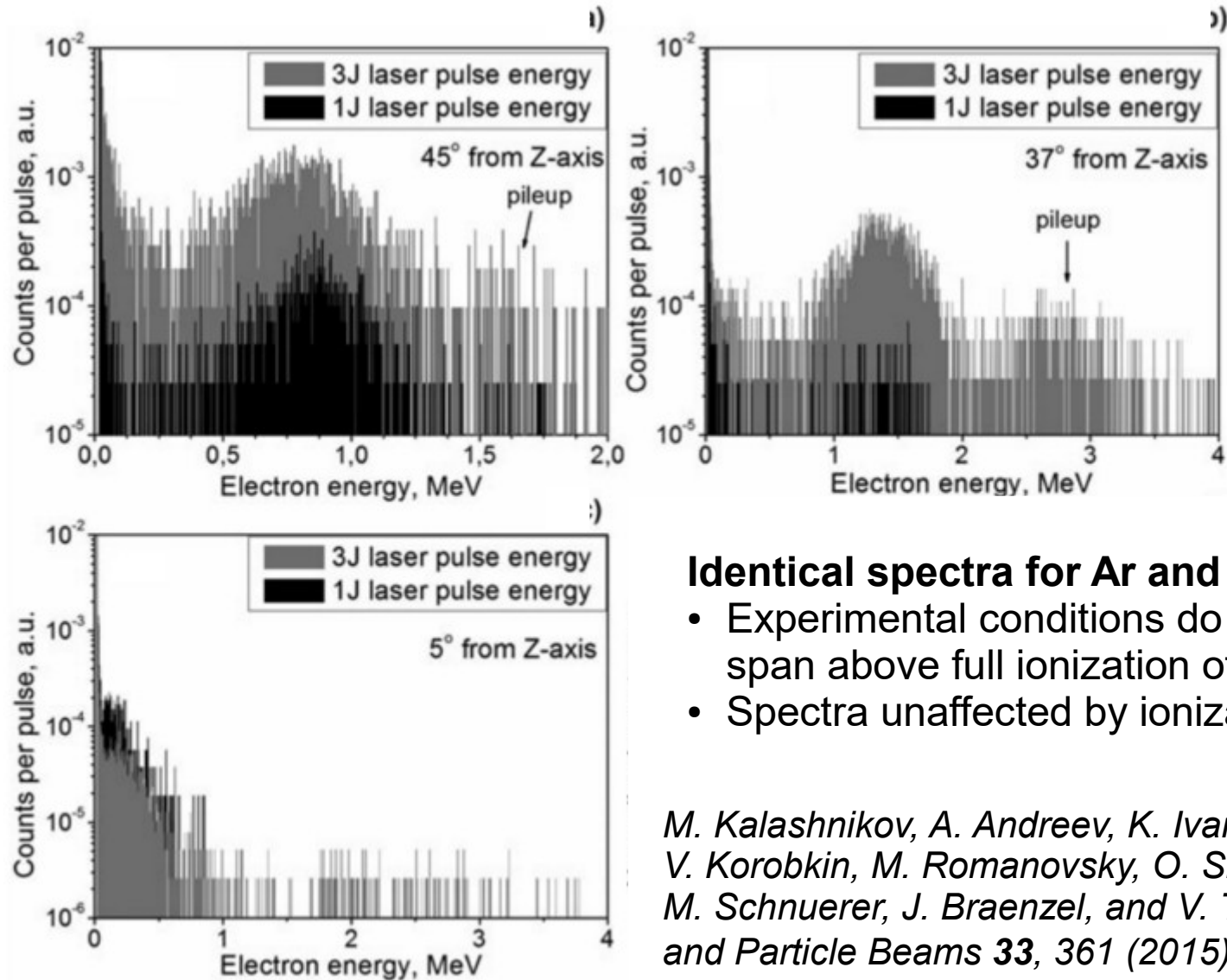
- Peak Intensity:  $>10^{20}$  W/cm<sup>2</sup>
- Pulse duration: 30 fs
- Focal spot size: 2.2 microns

#### Target:

- He, Ar, Kr
- Pressure:  $10^{-3}$  mbar
- Accumulation of energy spectra over multiple shots
- Ionization stage diagnostic: **ionization self-injection into focal spot ruled out**

*M. Kalashnikov, A. Andreev, K. Ivanov, A. Galkin, V. Korobkin, M. Romanovsky, O. Shiryayev, M. Schnuerer, J. Braenzel, and V. Trofimov, Laser and Particle Beams 33, 361 (2015)*

# Scattering of a Sparse Ensemble of Electrons: Energy Spectra



### Identical spectra for Ar and Kr:

- Experimental conditions do not span above full ionization of Ar
- Spectra unaffected by ionization

*M. Kalashnikov, A. Andreev, K. Ivanov, A. Galkin, V. Korobkin, M. Romanovsky, O. Shiryayev, M. Schnuerer, J. Braenzel, and V. Trofimov, Laser and Particle Beams 33, 361 (2015)*

## Relativistic Intensity, Highly Nonlinear Electron Dynamics

$$I_r = m^2 c^3 \omega^2 / 8 \pi e^2 = 1.37 \times 10^{18} (1/\lambda[\mu])^2 [\text{W/cm}^2]$$

## Implications of the Lawson-Woodworth Theorem

$$j = \sqrt{1 + p^2} - p_z = \text{const}$$

- Prohibits net energy gain in 1D
- Prescribes non-overlapping energy intervals with distinct angular ranges in 3D

**Electron energy spectra combining cold and relativistic components within a single angular range may not be explained on the basis of conventional theory**

## Electromagnetic Envelope in Vacuum: Groundwork for Theory

### Field:

$$\Delta \mathbf{A} - \partial_t^2 \mathbf{A} = 0, \quad (\nabla, \mathbf{A}) = 0.$$

$$\nabla = (\partial_x, \partial_y, \partial_z)$$

### Electrons:

$$\mathbf{p}_t = \mathbf{A}_t - \gamma^{-1} (\mathbf{p} \times (\nabla \times \mathbf{A}))$$

$$\gamma = \sqrt{1 + \mathbf{p}^2}$$

- Focused envelope
- Pulse-duration effects
- Calculating field to all orders in

$$\epsilon = (\lambda/2\pi w_0)$$

- All field components to be included
- Collective effects presumed to be negligible

## Asymptotic Framework

$\tau = 2\epsilon z$  - stretched variable

$s = t - z$  - co-moving variable

$\epsilon$  - asymptotic parameter

$w_0$  - focused pulse waist

$\lambda$  - laser wavelength

## Highly Nonlinear Regime

$$I_r = m^2 c^3 \omega^2 / 8 \pi e^2 = 1.37 \times 10^{18} (1/\lambda[\mu])^2 [\text{W/cm}^2]$$

## Asymptotic Series for Optical Envelope

$\tau = 2\epsilon z$  - stretched variable

$s = t - z$  - co-moving variable

$\epsilon = (\lambda/2\pi w_0)$  - asymptotic parameter

$w_0$  - focused pulse waist

$\lambda$  - laser wavelength

$$A_1 = \exp\left(i\frac{t-z}{\epsilon}\right) \left( a(\tau, x, y, s) + \sum_{m=1}^{\infty} \epsilon^m a_{1m}(\tau, x, y, s) \right) + c.c.$$

$$A_3 = \exp\left(i\frac{t-z}{\epsilon}\right) \sum_{m=1}^{\infty} \epsilon^m a_{3m}(\tau, x, y, s) + c.c.$$

## Asymptotic Series for Optical Envelope

$\tau = 2\epsilon z$  - stretched variable

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$\epsilon = (\lambda/2\pi w_0)$  - asymptotic parameter

$w_0$  - focused pulse waist

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**Corrections of same order as longitudinal field!**

$$A_1 = \exp\left(i\frac{t-z}{\epsilon}\right) \left( a(\tau, x, y, s) + \sum_{m=1}^{\infty} \epsilon^m a_{1m}(\tau, x, y, s) \right) + c.c.$$

Corrections known to be responsible for  
electron energy gain enhancement

$$A_3 = \exp\left(i\frac{t-z}{\epsilon}\right) \sum_{m=1}^{\infty} \epsilon^m a_{3m}(\tau, x, y, s) + c.c.$$

## Ground-State and First-Order Solutions for Optical Envelope

$\tau = 2\epsilon z$  - stretched variable

$s = t - z$  - co-moving variable

$\epsilon = (\lambda/2\pi w_0)$  - asymptotic parameter

$w_0$  - focused pulse waist

$\lambda$  - laser wavelength

$$-4i\partial_\tau a + \Delta_\perp a = 0$$

$$-4i\partial_\tau a_1 + \Delta_\perp a_1 = 4\partial_{\tau s}^2 a$$

$$\Delta_\perp = \partial_x^2 + \partial_y^2$$

**Duration-related corrections!**

$$a(\tau, x, y, s) = a_0(s)u(\tau, x, y)$$

$$a_{11}(\tau, x, y, s) = ia'_0(s)\partial_\tau (\tau u(\tau, x, y))$$

$$a_{31}(\tau, x, y, s) = -ia_0(s)\partial_x u(\tau, x, y)$$

$a_0(s)$  - optical field temporal profile

$u(\tau, x, y)$  - envelope solution



## Building Blocks: the Gaussian Pulse

$$u(\tau, x, y) = \frac{\Lambda(\tau, r)}{\sqrt{\tau^2 + 1}} \exp(i\psi(\tau, r))$$

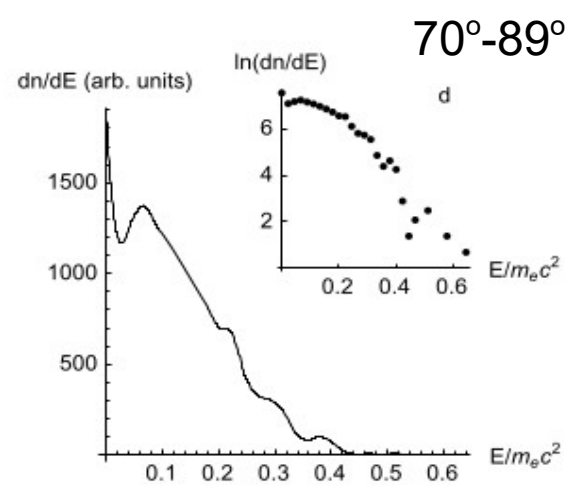
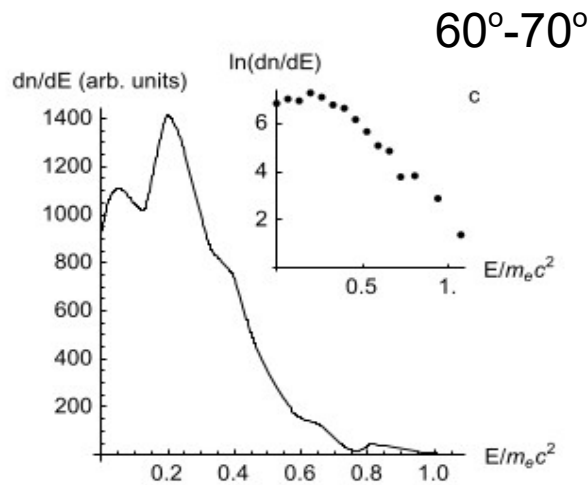
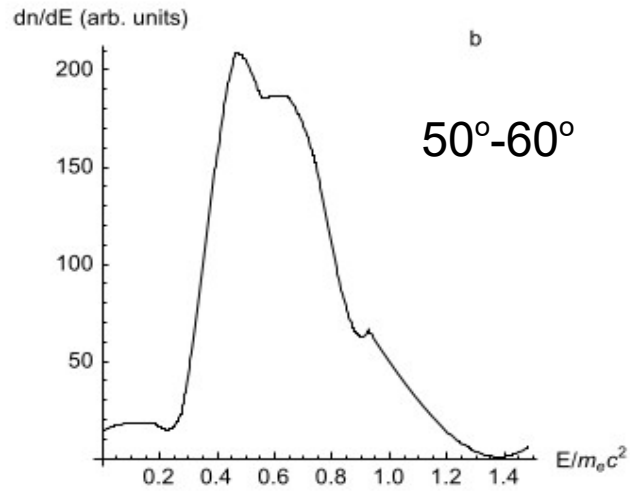
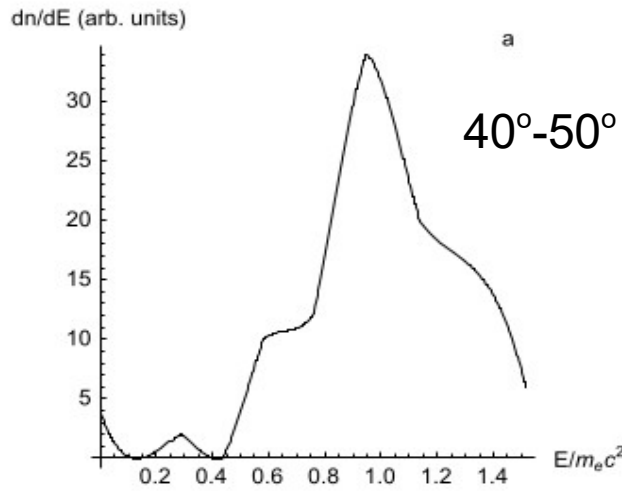
$$\Lambda(\tau, r) = \exp\left(-\frac{r^2}{\tau^2 + 1}\right), \quad \psi(\tau, r) = -\frac{\tau r^2}{\tau^2 + 1} + \arctan(\tau)$$

$$r = \sqrt{x^2 + y^2}.$$

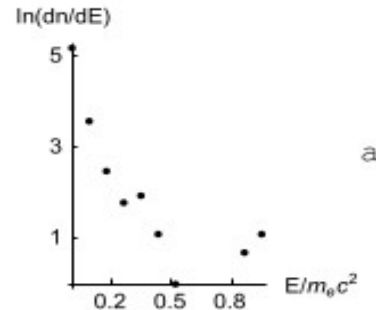
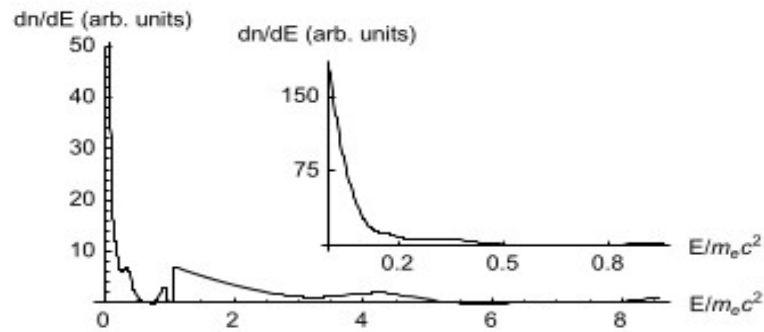
## Building Blocks: Laguerre Mode

$$u(\tau, x, y) = \frac{2^{l/2} \left(\frac{r}{\sqrt{\tau^2 + 1}}\right)^l \sin(l\varphi + \varphi_0) L_\delta^l\left(\frac{2r^2}{\tau^2 + 1}\right) \exp\left(i(2\delta + l + 1) \tan^{-1}(\tau) - \frac{r^2(1+i\tau)}{\tau^2 + 1}\right)}{\sqrt{\tau^2 + 1}}$$

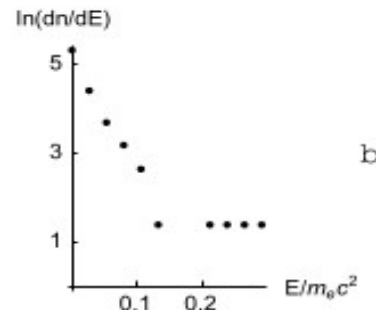
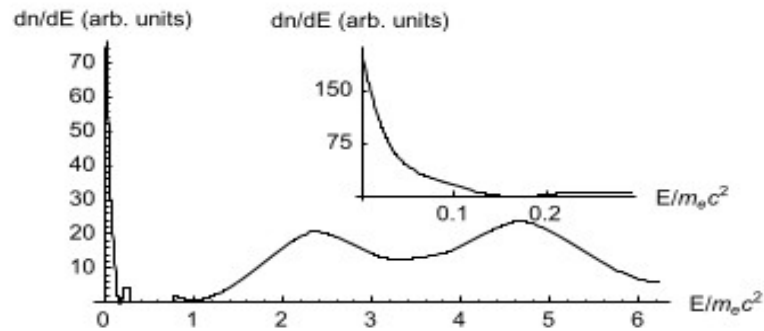
# Calculated Energy Spectra: Gaussian Pulse



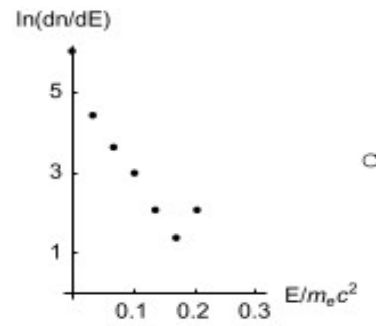
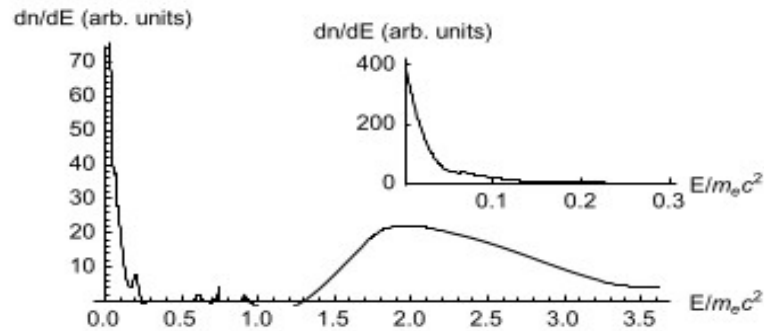
# Calculated Energy Spectra: Laguerre Mode



10°-20°



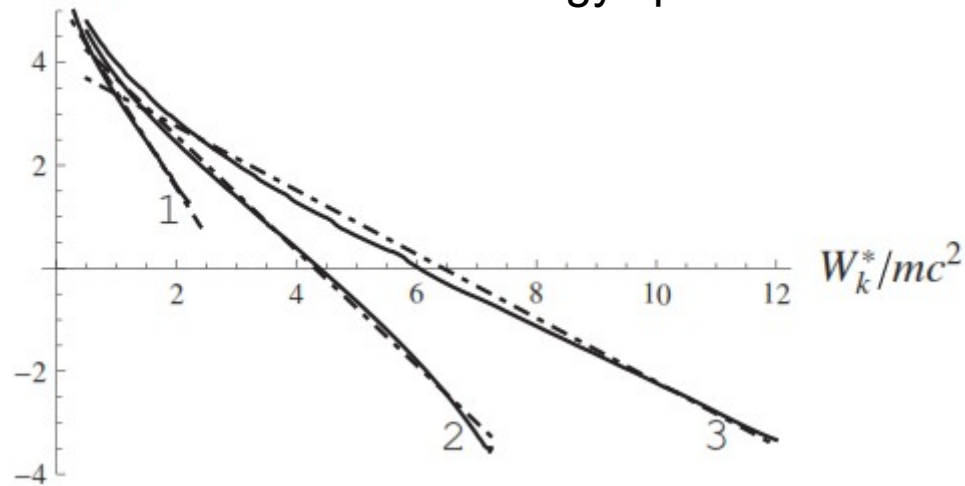
30°-40°



40°-50°

# Intensity Diagnostic Issues

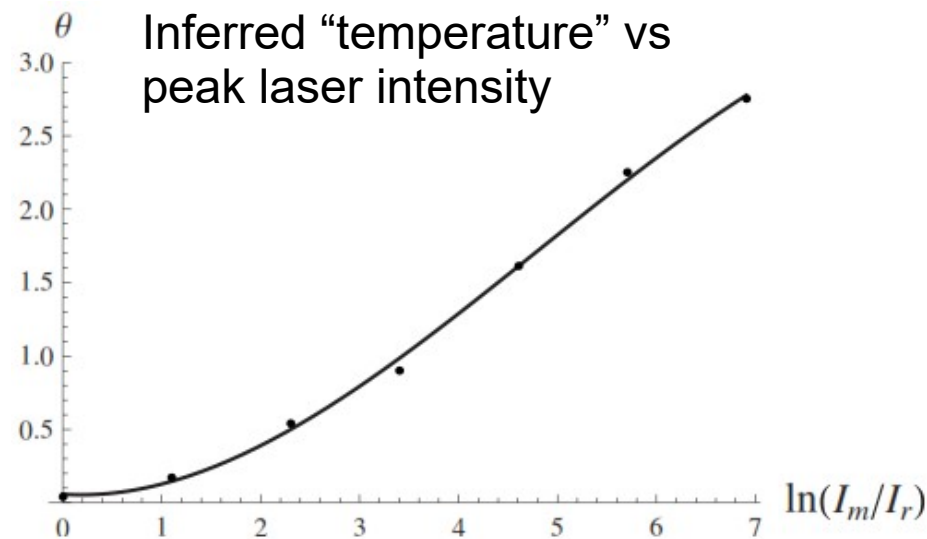
$\ln(P/P_0)$  Electron energy spectra



$$P/P_0 \sim \exp(-W_k/kT)$$

$$\theta = kT/mc^2$$

$$kT/mc^2 = 0.5 \ln(1 + 0.25 I_m/I_r)$$



Laser intensity:  $I_m/I_r = 10, 30, 100$

Pulse duration:  $c\tau/\lambda = 11$

Focal spot:  $\rho_0/\lambda = 8.5$

## Epilogue

Theory of focused optical envelope in vacuum requires substantial modification to take into account short-pulse effects within a unifying asymptotic framework. Short-pulse effect terms contribute to the electron scattering on a par with the terms describing the longitudinal component of the laser field

Neither the estimates based on adiabatic invariants nor simulations with Gaussian pulses explain the experimentally observed combination of cold and relativistic energies withing single angular ranges

The combination is attributable to the mingling impact of the stronger core and weaker ring in a Laguerre optical field mode

The applicability of laser intensity diagnostic techniques based on electron energy spectra is contingent on laser beam quality