DISTRIBUTION FUNCTION EVOLUTION OF ELECTRONS STOCHASTICALLY HEATED BY PICOSECOND LASER PULSE

L.A. Borisenko[1,2], N.G. Borisenko[1], A.M. Chekmarev[1], Yu.A. Mikhailov[1], **A.S. Orekhov**[1], A.A.Shapkin[1], G.V. Sklizkov[1]

[1] ***P.N. Lebedev Physical Institute RAS***, *Moscow, Russian* [2] *Moscow State University, Moscow, Russia*

Partly supported by RFBR 15-52-45116

In many works of recent the generation of electrons with anomalously high energy, considerably exceeding the equilibrium energy for In many works of recent the generation of electrons with anomalously high energy, considerably exceeding the equilibrium energy for the respective flux density [Nakamura, 2002] has been observed under the irradiation of solid targets by high-intensity laser radiation. In [Basov, 1982] for the first time the ion velocity, which corresponded to anomalously high energy up to a few MeV was discovered in experiments on the Doppler shift of the resonance lines of multiply charged ions in X-ray range at moderate intensity on target ~10^{14} W/cm². The presence of these ions also indicates the generation of high-energy electrons. One explanation for the occurrence of such electrons may be the mechanism of stochastic heating of charged particles in an electromagnetic field with random phase changes of the field resulting in the appearance of a random force acting on the electron during its motion [Mikhailov, 2010].

In the electromagnetic field of the light wave, the electron periodically is accelerating and slowing down, while it oscillates in the energy space and gains some energy of the oscillation, comparable with energy acquired during a quarter of the wave period. The electron energy is not much more equilibrium energy in laser plasma. Periodic variations of the electron energy occur much more slowly than the wave frequency, at least for relativistic motion of a particle in fields relevant flux density more than 10^{13} W/cm². If at the end of the acceleration period to change the local phase of the wave and consequently the phase of the Lorentz force acting on an electron, then there is a possibility of multiple acceleration. Ideally, it is desirable to change the phase synchronously with the movement of the electron. In reality, the change in the wave phase, which "sees" the electron occurs randomly.

A random stochastic effect on the electron can cause the following factors: fluctuations in the electromagnetic field due to a spatially inhomogeneous structure of multi-mode radiation focused on the target [Krylenko, 2011], plasma oscillations, which lead to the appearance of spontaneous electric fields in the longitudinal direction [Bochkarev], fluctuations the relative phase of the spectral components of the inhomogeneously broadened laser line, spontaneous magnetic fields [Lebo], as well as the fluctuations of the refractive index of the plasma using a low-density microstructured targets [Chaurasia, 2016].

In [Ivanov, 1995; 1996] by direct measurement of the emission current there were experimentally registered electrons with abnormally high energy in comparison with the thermal one. It has been found that a significant proportion (~10%) of the emitted electrons have an energy of 100 keV at a light flux density about 10^{13}-10^{14} W/cm², which corresponds to the temperature of the plasma is only about ~ 500 eV. So the proportion of these electrons is several orders of value higher than Maxwell one. In works [Mikhailov, 2008; 2010] attempted a numerical evaluation of the possibility of stochastic heating of the electron ensemble in the EM field of a plane wave and the qualitative comparison with experimental data. The temperature of the gas is estimated as the average energy of the corresponding non-equilibrium distribution function of relativistic electrons, averaged over the time of the laser pulse.

In the presented paper we consider the dynamics of electron emission depending on the structure of the laser pulse, which is represented a wave packet for the spectral composition of the neodymium laser radiation. Used in this case refers to the emission of electrons from their exit area EM field interaction , i.e. of the focus area. The distribution function of the flow of emitted electrons is of interest. The distribution function shape was obtained as in the energy and in the momentum representations. The dependence of the distribution function on time during the laser pulse at flux density up to 1018 W/cm2 has been calculated.

The formulas for electromagnetic fields with random parameters, leading to stochastic acceleration of particles are obtained. The possible sources of randomness in the laser-plasma system are analyzed.

The wave packet, modeling parameters of the laser field in the interaction region

The structure of the radiation line of a neodymium laser with inhomogeneous broadening of Stark components of the neodymium transition $^4F_{3/2}$ - $^4I_{11/2}$ and the processes of generation of short laser pulses in neodymium-glass has been thoroughly studied previously in many works [Ivanov, 1986; 1987; Senatsky, 2016]. In this paper, the simplified analytical model of a short laser pulse (wave packet) based on data of neodymium laser radiation, that is more convenient for numerical calculations has been developed.

Consider a number of assumptions of this model. The two upper Stark sublevels and 6 the lower sublevels are considered to be equidistant, their splitting is the same, although in reality the splitting of the upper level is almost 2 times greater than the distance between the lower sublevels of the $^4I_{11/2}$ (75 cm⁻¹). The twelve Stark components have the same frequency shape, which is close to Lorenz one, but they have different amplitudes.

A short laser pulse has a bell-shaped parabolic shape, i.e., a ideal contrast ratio. In the near zone the field intensity in the beam is supergaussian (8th order) close to a rectangular distribution over the aperture with a sharp boundary on the edges.

The field wave front is inhomogeneous, the field amplitude variation is random. The fluctuation of the field amplitude along the aperture is ~10%, and the field local phase varies randomly.

The spatial intensity distribution over the aperture has the form of a speckle structure. The size of the spatial inhomogeneity of the amplitude and phase of the order of several mm (spatial coherence), which corresponds to the observed. Therefore, the direction of the wave front locally changing randomly. All this has an impact on the distribution of the electromagnetic field focused onto a target. The amplitudes of the Stark components decrease as their distance from the main frequency symmetrically in contrast to the data of numerical calculations in articles mentioned above. In our model, the effects associated with cross relaxation, and the different effectivenesses of inversion depletion is not taken into account. All twelve Stark components of the radiation line are considered to be equivalent.

Formulas $f_i = \left(1-\frac{t_i}{\tau}\right)\cdot\frac{t_i}{\tau}$ and $fj_j = \left(1-\frac{j}{NL}\right)\cdot\frac{j}{NL}$ define the temporal pulse shape and distribution of components for the amplitude, respectively.

Here, index "*i*" is the current time index running through the values from 1 to $N = n_\lambda\, N_T$, where n_λ is the number of wave periods with the duration τ and N_T the number of points for a period. The index "*j*" is the number of laser line components. In this case, the composite wave consisting of 12 components can be written as $EL_i = \sum_j \left(fj_j \cdot f_i \cdot e^{i\omega_o\left(1+\frac{\Delta n_\lambda}{n_\lambda}\left(j-\frac{NL}{2}\right)\right)\left(t_i-\frac{\tau}{2}\right)} \right)$

The wave with random phases of components can be writen as $EL_{Rnd_i} = \sum_j \left(fe_{i,j}\cdot f_i\right)$, where $fe_{i,j} = fj_j \cdot e^{i\left(\omega_o\left(1+\frac{\Delta n_\lambda}{n_\lambda}\left(j-\frac{NL}{2}\right)\right)\left(t_i-\frac{\tau}{2}\right)-\Phi_j\right)}$, where the function of time Φ_j is random phase of a component.

In Fig. 1 on the left the frequency spectra of the components with the random phases Φ_j are shown. On the X axis is the frequency divided by ***c*** - the speed of light. Center frequency equaled to $\omega_0 = \frac{2\pi}{\lambda_0} = 5.928\cdot 10^4$ s⁻¹ corresponds to the wavelength of the neodymium in silica matrix $\lambda_0 = 1.06$ μ. Given a family of curves is the real part $R_{p,j} = \mathrm{Re}\left(ELj_{\omega Rnd_{p,j}}\right)$ of the complex amplitude $ELj_{\omega Rnd_{p,j}} = \sum_i \left[fe_{i,j}\cdot f_i \cdot e^{-i\omega_p(t_i-0.5\tau)}\cdot \Delta t\right]$, where the frequency in the vicinity of the line $\omega_p = 0.99\omega_o + \frac{0.02\omega_o}{pp}\cdot p$, and the index "***p***" runs through the values (0 ... *pp*=100) in the frequency range of $\Delta\omega = 0.119\cdot 10^4$s-1.

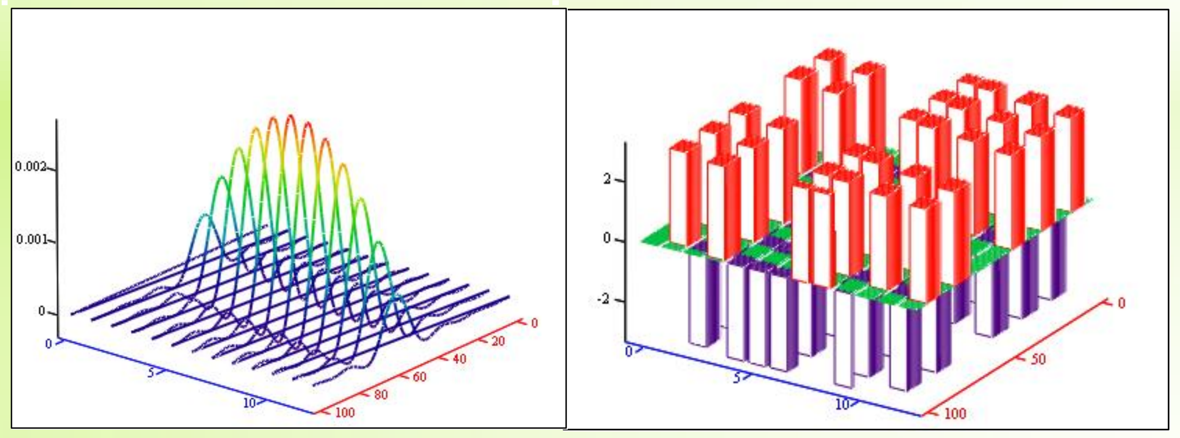


Fig. 1. Spectral components of laser radiation field based on the random phase. On the left along X axis - frequency, along Y axis - energy of a component. On the right - the phase structure of spectral components.

The total field of all components with external to the wave packet phase ψ_i can be written as $EL_{Rnd\psi_i} = \left(\sum_j \left(fe_{i,j}\cdot f_i\right)\right)\cdot e^{i\psi_i}$. A few words about the random function ψ_i. The index "*i*" corresponds to the temporal coordinate. The external phase is the field phase, which is "seeing by an electron". It determined by the relative position of the electron and the wave field. In other words, it is a local phase of the field at the location of the moving electron at a given point and at a given time. The phase ψ_i depends on the spatial structure of the field, because of the electron usually relativistic, moves through space partially randomly changing along with the position and velocity as well as the properties of the wave field. The process of interaction of the electron with the field in the present case, takes place quite a long time, i.e. over many periods (tens or hundreds). This is so-called stochastic acceleration. In contrast to the direct acceleration in one or more periods in the case of using in fact several independent lasers.

Therefore, in our case, the random variation of phase in time is also significant. Thus the form of this function, that depend on time and coordinates, in fact models the random nature of the force acting on the electron. As the sources of randomness, along with the spatial and temporal structure of the EM field, can be microstructure of plasma, which is formed by laser heating of structured low-density foamed targets, the plasma waves generating random longitudinal electric fields, Alfvén waves if heating occurs in the presence of strong microscopic magnetic fields, as well as intrinsic spontaneous magnetic fields.

Fig.2 shows a typical amplitude of wave field and the local phase at the location of an electron. The curves correspond to a time interval of duration about 1.5·10⁻³ ps in the middle of 2.12 ps pulse. The whole time at the X-axis equevalent to 22 periods of central wave with a period of 3.533 fs. The laser pulse is bell-shaped with a perfect contrast in intensity. The amplitude of the phase change ψi is given with uncertainty up to a whole number of periods. This function describes the sudden random change of the motion of a charged particle (velocity and direction).

This is most significant for modeling of phase change in the moment of interaction in the case of relativistic velocities. In the system associated with the electron we are dealing in fact with a random change of direction of the wave vector. The periodic step function ψi (purple curve in Fig.2) is set for the modeling of random disturbances of the force acting on electron. It has a randomly changing and constant within a period amplitude. Variation in the amplitude is selected like "white noise". The frequency of change is a constant function and it is chosen in accordance with implied mechanism of the perturbation of an electron motion.

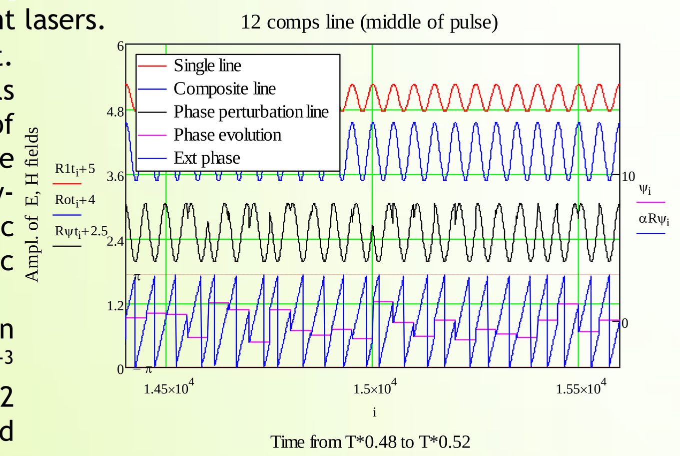
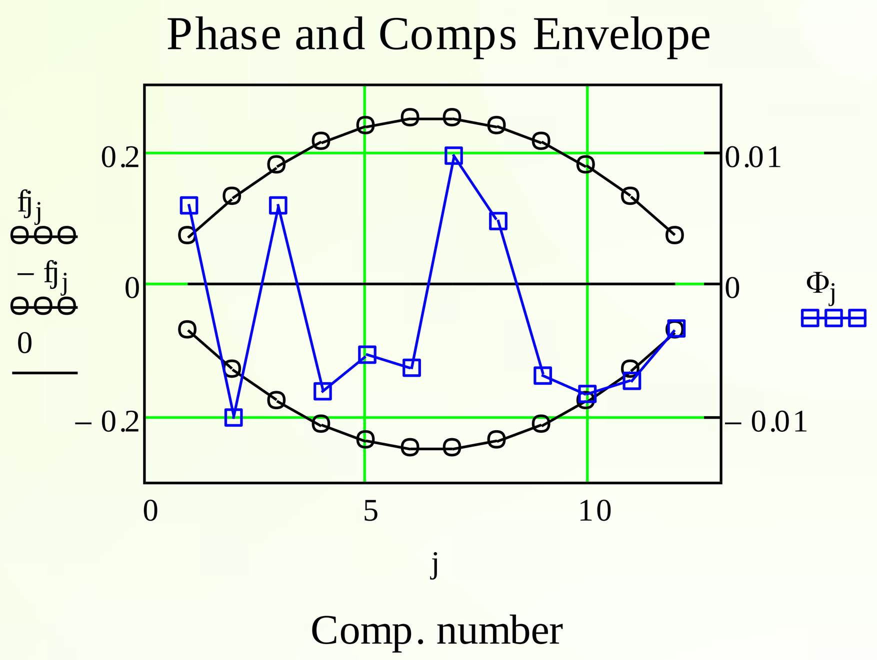


Fig.2. The usual amplitude of the wave and the local phase at the point of finding the electron. The upper curve corresponds to a plane wave sinusoid (**Rlt**$_i$) without phase perturbation. The second from the top - composite line (**Rot**$_i$) without phase perturbation. The third curve - the total amplitude of the composite wave (**Rψt**$_i$) with considering the random phases **Φ**$_j$ and **ψ**$_i$. The purple step curve at the bottom - random phase **ψ**$_i$ with constant average period of perturbation. The blue curve at the bottom (**αRψ**$_i$) - phase of composite wave field at the point of electron location at a given time.

Fig.3 shows the phase pattern for components in a given time. The phase change of each component in time occurs independently of each other. As the set Random phase change of the field of each component is selected as a given, i.e. it does not depend on the state of the ensemble of electrons. The envelope of the maxima of the field components has a bell-shape form.

The phase of the field acting on the electron changes randomly in time and frequency, Fig.4. Visible asymmetry in the amplitude of phase shift on frequency especially in the wings of the line is evident. It is assumed that the phase jump occurs when the direction of the field vector due to an abrupt change of direction of motion of the electron. The latest is simulated by the random function ψ_i.



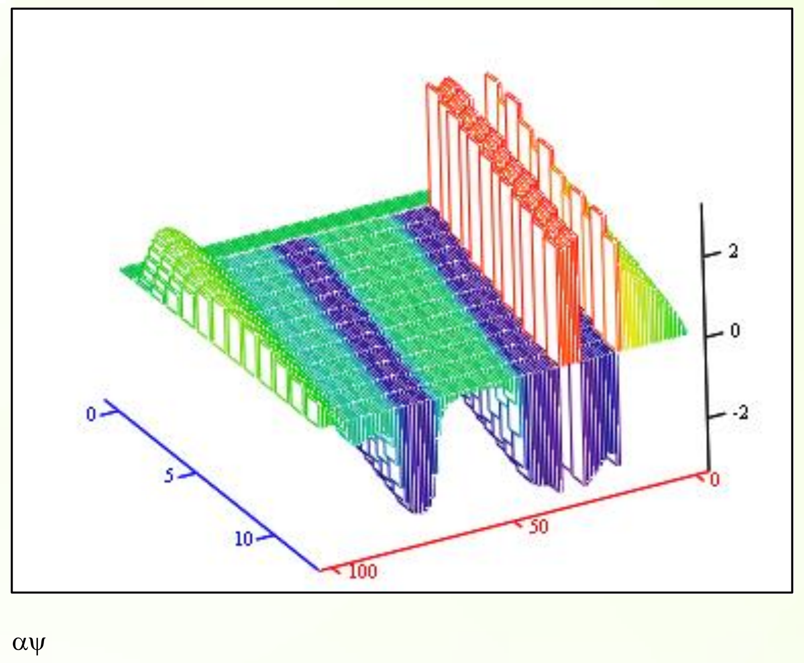
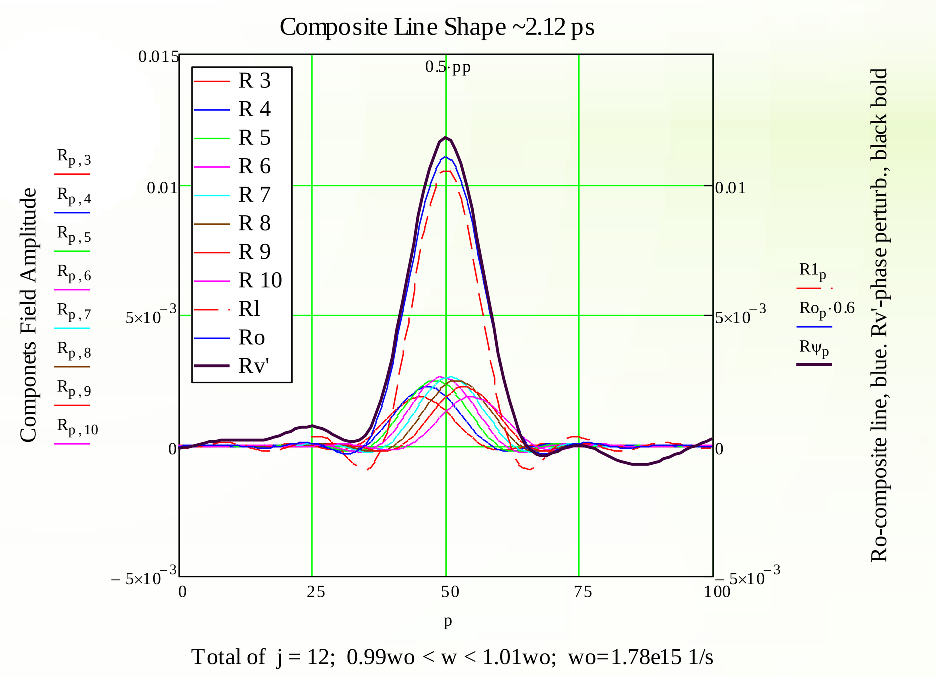


Fig.3. The phase pattern of components at a given moment. Round dots show the maximum amplitude of the components in the relative units (left axis). Square dots - random phase of each component in radians (right axis). Index "**j**" - component number.

Fig.4. The instantaneous random distribution pattern of phase frequency for different components. On the blue Y axis is component number. On the red X axis is frequency.

Fig.5 shows the structure of the laser line with random phases for the neodymium laser with silicate glass with a wavelength $\lambda = 1.06$ μ. Curves are given for the electric field strength in the wave. The black bold curve illustrates the instantaneous shape of a line, depending on the frequency when taking into account the random phases of all components and function ψ_i . The asymmetry of both phase and amplitude in the line wings relative to the central frequency $\omega_o = 1.78\cdot 10^{15}$ s⁻¹is visible. A small variation of the frequency at the maximum is also present (not shown). The spectral lines without disturbing phases for 12 components (dotted red curve) is also shown for comparison. There is a complete symmetry. All the curves in the figure are for the duration of the pulse 2.12 ps, i.e., 600 oscillation periods of the field.

Fig.5. The structure of the laser lines with random phases, consisting of 12 components (bold black curve). Thin lines - 8 components (the weak lateral components are not shown). The dotted red curve - the component line consisting of the 12 components without disturbing phases.



Electron motion equation

The equations of motion of an electron can be written as

$$\begin{cases} \dfrac{d\vec{r}}{dt} = \vec{\beta}(\vec{p}), \quad \vec{\beta}(\vec{p}) = \dfrac{\vec{p}}{\sqrt{1+(\vec{p}\cdot\vec{p})}}, \\ \dfrac{d\vec{p}}{dt} = q\cdot\vec{\beta}(\vec{p})\times\vec{B}(\vec{r},t) + q\cdot\vec{E}(\vec{r},t) - L(\vec{r}), \end{cases},$$

The variables following $t = ct'$; $\bar{p} = \frac{\bar{p}'c}{mc^2}$; $T_{kin} = \frac{T'_{kin}}{mc^2}$; $\bar{\beta} = \frac{\bar{v}'}{c}$; mean reduced values of time, momentum, kinetic energy and velocity respectively. The values with a prime relate to real ones. The last term $L(\vec{r})$ in the second equation describes the specific radiation losses averaged per laser pulse.

The electromagnetic field is considered as a predetermined, that is it does not depended on the properties of a plasma. Based on the above, in the preceding section, the wave fields with considering random phases can be written as following

$$\vec{E}(\vec{r},t) = \vec{E}_o \cdot EL_{Rnd\psi}(t)\, e^{i\omega_o t}\cdot\left[(1-\gamma_r)e^{-\vec{k}\vec{r}} + 2\gamma_r\cos(\vec{k}\vec{r}+\phi(\vec{r}))\right];$$

$$\vec{B}(\vec{r},t) = \mathbf{\vec{B}}_o \cdot EL_{Rnd\psi}(t)\, e^{i\omega_o t}\cdot\left[(1-\gamma_r)e^{-\vec{k}\vec{r}} + e^{-\frac{\pi}{2}}2\gamma_r\sin(\vec{k}\vec{r}+\phi(\vec{r}))\right].$$

Here γ_r - amplitude reflection coefficient; $\vec{k}$ - wave vector; ϕ - reflection wave random phase. The reflected wave is formed near critical density. In this model it is assumed that the reflected wave has the same structure as the incident wave, i.e. specular reflection from a rough surface. This assumption makes sense only for small coefficients of reflection that is implemented in most laser experiments.

Electron motion

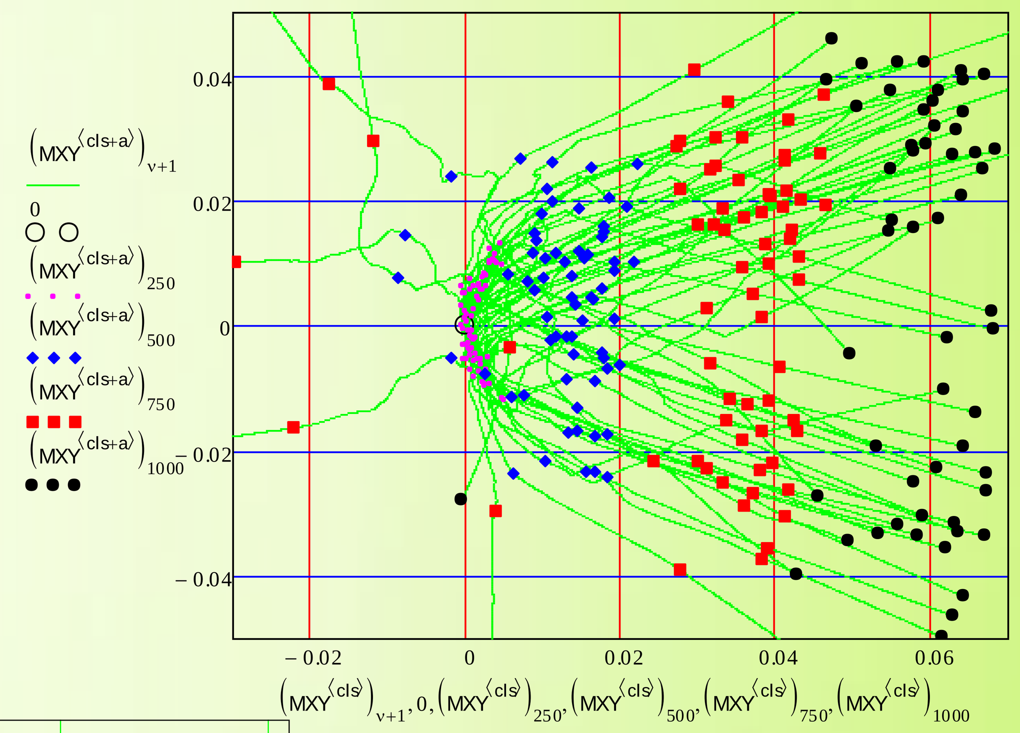


Fig. 6. Typical electron trajectories are shown by green curves. The pulse shape of the heating laser parabolic bell at length on the base 3.5 ps. The radiation intensity in interaction area is about 2·10¹⁷ W/cm². The dots show the position of the electrons at different times. From left to right: 0.875 ps (purple dots); 1.75 ps (blue diamonds); 2.625 ps (red squares); 3.5 ps (black circles). Radiation propagation direction is parallel to the axis X. The polarization vector is parallel to the axis Y. There is a cone-shaped expansion of the accelerated electrons with a divergence of approximately 0.35 rad.

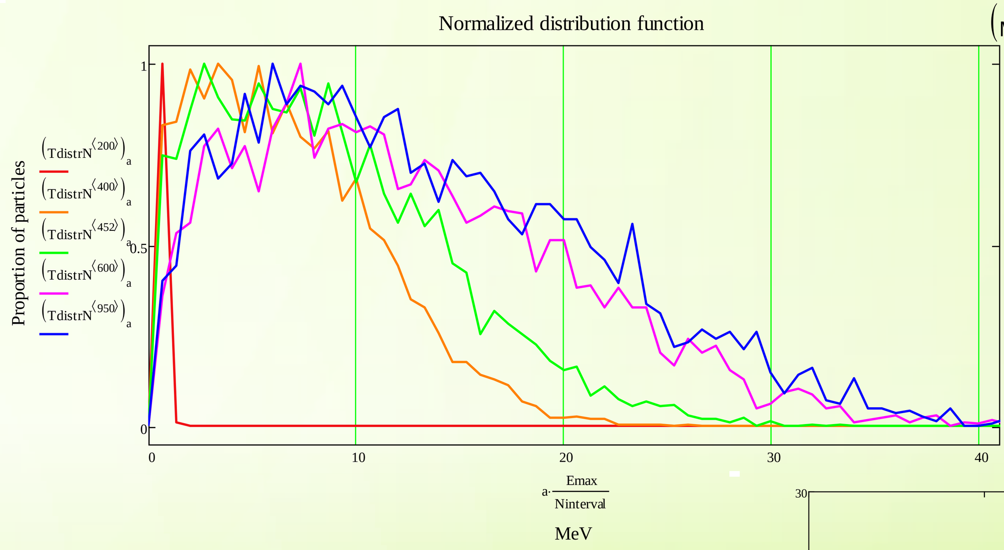


Fig. 7. The evolution of normalized probability density of electron distribution function over time. The kinetic energy of electrons is plotted along the X-axis in MeV.

Curve No	200	400	452	600	950
Time corresponding, ps	0.7	1.4	1.58	2.1	3.5

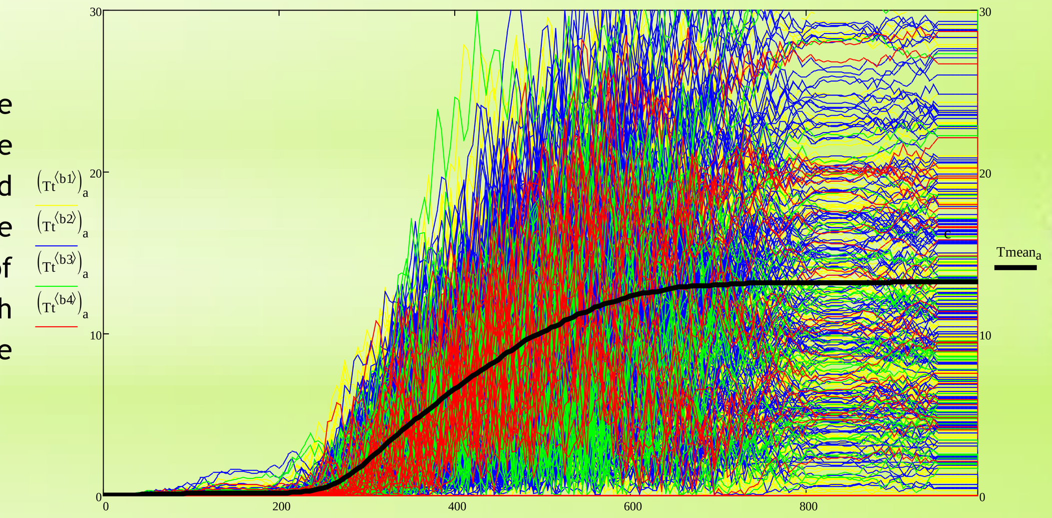


Fig. 8. The black bold curve illustrates the dependence of electron kinetic energy on time within pulse duration of 3.5 ps. The energy of electrons is averaged over 4000 electrons at each moment. It is seen the delay of development of stochastic heating of electrons. For the field strength of eb=30 which corresponds to laser flux density ~2·10¹⁷ W/cm² the delay is about 1 ps.

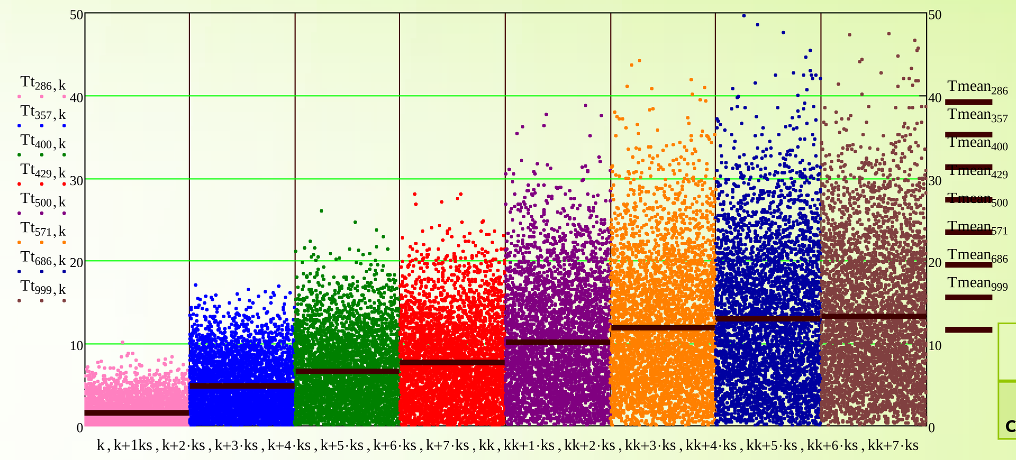


Fig. 9. Energy of an electron at a given moment. Bell shaped total pulse duration is 3.5ps. Bold black lines show energy of electrons averaged over all particles for corresponding instants mentioned above. Colored dots show the distribution of electron energy for the same time points.

Curve No	286	357	400	429	500	571	686	999
Time corresponding, ps	1	1.25	1.4	1.5	1.75	2	2.4	3.5

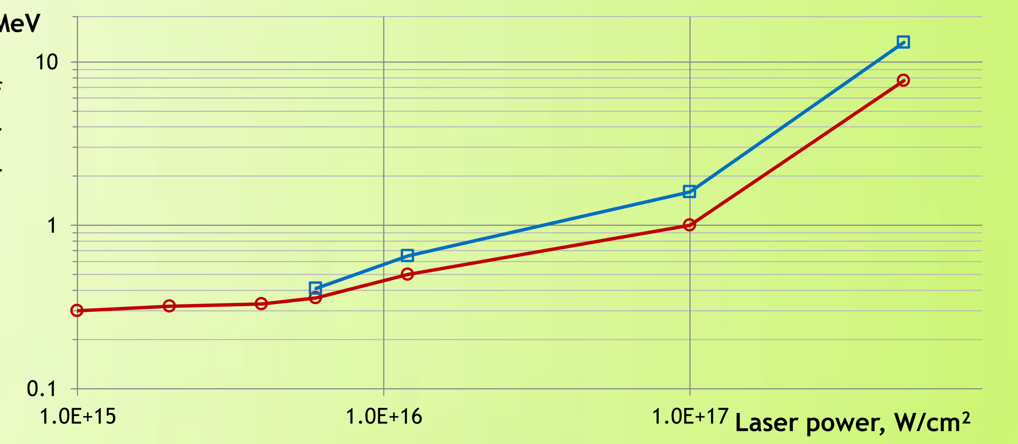


Fig. 10. The average energy of electrons at the end of the pulse (boxes). The electron energy averaged over all electrons and averaged over time of heating for each electron (circles).

Referrences

T.Nakamura, S.Kato, M.Tamimoto, T.Kato, "Stochastic acceleration by intense laser fields", Physics of Plasmas, (May 2002), vol. 8, No 5, pp

N.G.Basov, S.V.Bobashov, K.Gotz, M.P.Kalashnikov, A.P.Meshcherkin, Yu.A.Mikhailov, A.V.Rode, G.V.Sklizkov, S.I.Fedotov, E.Forster, and H.Endert, "Spectroscopic observation of MeV-range multiply charged ions in a laser plasma", Journal of Experimental and Theoretical Physics Letters (JETP Letters) 1982г., vol 36, № 7, pages 281-284.

Yu.V.Krylenko, Yu.A.Mikhailov, A.S.Orekhov, G.V.Sklizkov, A.A.Filippov, "The dependence of stochastically heated electrons on laser radiation flux density focused onto a target" Bulletin of the Lebedev Physical Institute, N10 (2010)

Yu.V. Krylenko, Yu.A. Mikhailov, A.S. Orekhov, G.V. Sklizkov, and A.M. Chekmarev, "Structure of a laser field of various polarizations in the focal region of an ideal focusing lens. Calculation by methods of scalar diffraction theory", Journal of Russian Laser Research, Volume 32, Number 1, pp 19-40, January, (2011)

Yu.A.Mikhailov, L.A.Nikitina, G.V.Sklizkov, A.N.Starodub, and M.A.Zhurovich. "Stochastic heating of electrons in focused multimode laser fields" Journal of Russian Laser Research, Vol. 28, No 4, (2007) pp 344-355

Yu.A. Mikhailov, L.A.Nikitina, G.V. Sklizkov, A. N. Starodub, and M.A.Zhurovich, "Relativistic electron heating in focused multimode laser fields with stochastic phase perturbations", Laser and Particle Beams, vol. 26, , pp. 525-536, (2008)

S.G.Bochkarev, A.V.Brantov, V.Yu.Bychenkov, D.V.Torshin, V.F.Kovalev, G.V.Baidin, and V.A.Lykov, "Stochastic Electron Acceleration in Plasma Waves Driven by a High_Power Subpicosecond Laser Pulse", Plasma Physics Reports, 2014, Vol. 40, No. 3, pp. 202-214.

V.V.Ivanov, A.K.Knyazev, A.V.Kutsenko, A.A.Matsveiko, Yu.A.Mikhailov, V.P.Osetrov, A.I.Popov, G.V.Sklizkov, and A.N.Starodub, "Investigation of the generation of high-energy electrons in a laser plasma", Zh. Eksp. Teo.r. Fiz., vol. 109, pp.1257-1266 (April 1996)

V.V.Ivanov, Yu.V.Senatsky, G.V.Sklizkov, "The impact of non-radiative relaxation at the level of the laser transition of Nd3+ ions in the glass on the amplification of powerful nanosecond pulses", QE, 1986, vol. 16, No. 3, pp. 422-424.

Yu.V.Senatsky, "Fundamental processes and the methods of formation of the light beams with solid state laser for plasma heating", Lebedev Physical Institute, Moscow, 2016.

Chaurasia, S.; Leshma, P.; Murali, C. G.; et al. "Studies on subcritical and overcritical density laser ablated TAC foam targets" Optics Communications, Vol. 343, Pp 1-5, MAY 15 2015