Can laser induced relativistic shock waves make a difference?

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1-Introduction
Who needs shock waves?

- From the time when Hugoniot completed the theory of shock waves in 1887 this subject is active in many fields of science:

1. Condensed matter physics (including material strength).
2. Astrophysics (supernova, neutron stars etc.).
5. Particle physics (quark-gluon matter).
7. Inertial confinement fusion (ICF).
8. etc., etc., etc.
Why SHOCK WAVES?

I. **static experiments:**
   The sample is squeezed between pistons or anvils. Pressure is limited by the strength of the materials.
   (maximum $P \sim 5\text{Mbars}$)

II. **dynamic experiments:** shock waves are created.
   Passage time of the shock is short in comparison with the disassembly time of the material.
   One can do shock wave research for any pressure that can be supplied by a driver, assuming that proper diagnostics are available.
Laser induced shock-waves


- In 1974 (Garching-Germany) the first direct observation of a laser-driven shock wave was reported: 2 Mbar.

- In 1994 Livermore-USA created a pressure of ~1Gbar (Indirect drive).

- In 2005 Osaka-Japan created a pressure of ~1Gbar (Direct drive).

RELATIVISTIC SHOCK WAVES

With $10^{24}$ W/cm$^2$ we expect pressures $P \sim 1 \text{ Pbar}=10^{15}$ bar
Hugoniot and thermodynamic curves


**Legend:**
- $H$ = Hugoniot
- $H_k$ = Hugoniots
- $S$ = Isentrope
- c.p. = critical point

**Diagram:**
- ICF
- $(1\text{Gb})$
- Solid
- Liquid
- Plasma
- Vapour
- $T=0$
- $c.p. (~10\text{kb})$
- $L+V$
- $V=\frac{1}{\rho}$
2- Micro-foil acceleration
• Transferring momentum from light to macroscopic objects:
P. N. Lebedev, Ann. Der Physik, vol 6, 433 (1901)

• Interstellar vehicle propelled by terrestrial laser beam
G. Marx, Nature 211, 22 (1966)

• Acceleration of foils by ultra-intense lasers for generating relativistic ions.

• Relativistic Rayleigh –Taylor instability.

• S. Eliezer, J. M. Martinez Val, (2012 and 2013)
Notation: Subscript F for foil rest frame; No subscript for laboratory frame.

The laser irradiance $I$:

$$I = I_F \left( \frac{\omega}{\omega_F} \right)^2 = I_F \left( \frac{1 + \beta_f}{1 - \beta_f} \right)$$

The electromagnetic pressure $P$:

$$P = P_F = \frac{I_F}{c} \left( 1 + R_F - T_F \right) = \frac{2I_F R_F}{c}$$

$$R_F + T_F = 1$$

$$R_F = 1 \Rightarrow P = \frac{2I}{c} \left( \frac{1 - \beta_f}{1 + \beta_f} \right)$$
Momentum of the microfoil: \( p_f = M_0 \gamma_f \beta_f c \), rest mass: \( M_0 = \rho_0 Sl \)
\( \rho_0 = \) initial density, \( S = \) cross section area, \( l = \) the thickness of the microfoil

\[
\frac{dp_f}{dt} = PS \quad \Rightarrow \quad \frac{d}{dt} \left[ (\rho_0 lc) \frac{\beta_f}{\sqrt{1 - \beta_f^2}} \right] = \frac{2I}{c} \left( \frac{1 - \beta_f}{1 + \beta_f} \right)
\]

\[
\Rightarrow \quad \frac{1}{(1 + \beta_f)^{1/2} (1 - \beta_f)^{5/2}} \frac{d\beta_f}{dt} = \frac{2I}{\rho_0 c^2 l}
\]
\[ I = \text{const} \rightarrow \int_0^{\beta_f} \frac{dx}{(1-x)^{3/2}(1+x)^{1/2}} = \frac{(2-\beta_f)\sqrt{1-\beta_f^2}}{3(1-\beta_f)^2} - \frac{2}{3} = \frac{2It}{\rho_0c^2l} \equiv \frac{t}{\tau} \]

\[ W_L = ItS = \frac{\rho_0c^2Sl}{2} \left( \frac{t}{\tau} \right) = \left( \frac{\rho_0c^2Sl}{2} \right) \left[ \frac{(2-\beta_f)\sqrt{1-\beta_f^2}}{3(1-\beta_f)^2} - \frac{2}{3} \right] \]

Micro-foil velocity as function of the laser pulse duration

\[
\begin{align*}
\beta_f(t/\tau) \rightarrow 1 & \quad \text{for } t/\tau \rightarrow \infty \\
\text{laser pulse duration} & \\
\end{align*}
\]

\[
\begin{align*}
\text{t=}\tau_L = \tau & \quad \rightarrow \beta_f \approx 0.5 \\
laser pulse duration & \\
\end{align*}
\]

\[
\begin{align*}
t=\tau_L = \tau = \frac{\rho_0c^2l}{(2I)} & \quad \rightarrow \quad I_L \tau_L = \frac{\rho_0c^2l}{2} \\
\end{align*}
\]

\[
\begin{align*}
\beta_f \rightarrow 1 & \quad \text{for } t/\tau \rightarrow \infty \\
\text{laser pulse duration} & \\
\end{align*}
\]

\[
\begin{align*}
\text{t=}\tau_L = \tau = \frac{\rho_0c^2l}{(2I)} & \quad \rightarrow \quad I_L \tau_L = 4.5 \times 10^8 \text{ J/cm}^2 \quad \text{for } \rho_0 = 1 \text{ g/cm}^3 \\
W_L = I_L \tau_L S & = 45 \text{ J for } S = 10 \mu\text{m}^2 \\
\end{align*}
\]
**kinetic energy of the foil** $W_{Kf}$

\[
\frac{W_{Kf}}{S} = \rho_0 l c^2 \left( \frac{1}{\sqrt{1 - \beta_f^2}} - 1 \right); \quad W_L \equiv W_L (\text{in}) \geq W_{Kf};
\]

$W_L (\text{in}) > W_L (\text{reflected})$ also for $R_F = 1$ due to Doppler shift!

Non-relativistic limit $W_{Kf}/S \sim \beta_f^2 \sim t^2$ for $t \to 0$.

Relativistic limit $W_{Kf}/S \sim t^{1/3}$ for $t \to \infty$

**Kinetic energy per atom** $\varepsilon_{Kf} = W_{Kf}/N$

**Example (Al):**

$\rho_0 = 2.7 \text{ g/cm}^3, \, l = 0.1 \mu\text{m}, \, S = 10 \mu\text{m}^2$

$\rightarrow N(\text{Al atoms}) = 6 \cdot 10^{10}$

$\beta = 0.99 \quad \rightarrow \quad W_{Kf} \approx 1500J, \quad \varepsilon_{Kf} \approx 150\text{GeV}$

$\beta = 0.9 \quad \rightarrow \quad W_{Kf} \approx 300J, \quad \varepsilon_{Kf} \approx 30\text{GeV}$

$\beta = 0.5 \quad \rightarrow \quad W_{Kf} \approx 40J, \quad \varepsilon_{Kf} \approx 4\text{GeV}$

Note: $m_{\text{Al}}c^2 \approx 27m_pc^2 \approx 27\text{GeV}$
Collisions between 2 accelerated foils

\[ W_L = ItS = \frac{\rho_0 c^2 S l}{2} \left( \frac{t}{\tau} \right) = \left( \frac{\rho_0 c^2 S l}{2} \right) \left[ \frac{(2 - \beta_f)\sqrt{1 - \beta_f^2}}{3(1 - \beta_f)^2} - \frac{2}{3} \right] \]
Relativistic Rayleigh-Taylor (RT) instability

Non-relativistic RT: \[ \xi_{NR} = \Delta x / x_0 = \exp \left( t / \tau_{NR} \right) \]

Relativistic RT: \[ \xi_R = \Delta x / x_0 = \exp \left[ \left( t / \tau_R \right)^{1/3} \right] \]

\[ \tau_{NR} = \left[ \left( \frac{1}{8 \pi} \frac{\rho_0 \ell_x \ell_{yz} c}{I_L} \right) \right]^{1/2} \]
\[ \tau_R = \left[ \left( \frac{1}{72 \pi^3} \frac{\ell_{yz}^3 I_L}{\rho_0 \ell_x c^5} \right) \right]^{1/2} \]

foil \[
\begin{align*}
\ell_x &= \text{thickness}=0.1 \mu m; \ell_{yz} = y-z \text{ dimension}=10 \mu m \\
\rho_0 &= 1 g / cm^3; x_0 = \text{initial disturbance amplitude}=10 \text{nm}
\end{align*}
\]
laser: \( I_L = 10^{24} \text{W} / \text{cm}^2 \)

foil breaks \( \xi \sim 10 \Rightarrow \)
\[ NR: \text{foil breaks at } t=2.5 \text{ fs} \]
\[ R: \text{foil breaks at } t=50 \text{ fs} \]
3- Relativistic Shock Waves
The formalism

4-velocity: \( U_{\mu} = (\gamma c, \gamma v_1, \gamma v_2, \gamma v_3) \);

\( P = \) pressure, \( e = \) energy/volume (including mass energy), \( n = \) number of particles/volume

Metric tensor \( g_{\mu\nu} : g_{00} = -1, g_{11} = g_{22} = g_{33} = 1, g_{\mu\nu} = 0 \) if \( \mu \neq \nu \).

\( \gamma = \frac{1}{\sqrt{1 - \beta^2}} \); \( \beta = \frac{v}{c} \); \( c = \) speed of light; Lorentz transformation:

\[
\begin{pmatrix}
\gamma & -\gamma\beta & 0 & 0 \\
-\gamma\beta & \beta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{pmatrix}
\]

Energy-momentum 4-tensor:

\[
T_{\mu\nu} = (e + P)U_{\mu}U_{\nu} + Pg_{\mu\nu}
\]

Energy-momentum conservation:

\[
\frac{\partial T_{\mu\nu}^\nu}{\partial x^\nu} = \partial_{\nu} T_{\mu}^\nu = 0 \quad \text{for} \; \mu = 0, 1, 2, 3.
\]

Particle number conservation:

\[
\frac{\partial (nU^{\mu})}{\partial x^{\mu}} \equiv \partial_{\mu} (nU^{\mu}) = 0
\]

The equation of State:

\( P = P(e, n) \)

Note: Einstein summation is assumed (from 0 to 3) for identical indexes.
Conservation laws in the shock-wave (SW) reference system:

indexes 0 & 1 before and after SW singularity accordingly;

\[ \gamma_i = \frac{1}{\sqrt{1 - \beta_i^2}} \quad ; \quad \beta_i = \frac{v_i}{c} \quad ; \quad i=0 \text{ or } 1 \quad ; \quad c = \text{speed of light}; \]

Conservation of energy:

\[ \gamma_0^2 \beta_0 (e_0 + P_0) = \gamma_1^2 \beta_1 (e_1 + P_1) \]

Conservation of momentum:

\[ \gamma_0^2 \beta_0^2 (e_0 + P_0) + P_0 = \gamma_1^2 \beta_1^2 (e_1 + P_1) + P_1 \]

Conservation of number of particles:

\[ \gamma_0 \beta_0 n_0 = \gamma_1 \beta_1 n_1 \]

\( P, e, n, \rho = Mn, T \) defined in the rest frame of reference (of the fluid).

The solution (A. H. Taub, PR 74, 328 (1948):
Solution in the laboratory frame of reference

\[
\frac{u_s}{c} = \sqrt{\frac{(P_1 - P_0)(e_1 + P_0)}{(e_1 - e_0)(e_0 + P_1)}}
\]

\[
\frac{u_p}{c} = \sqrt{\frac{(P_1 - P_0)(e_1 - e_0)}{(e_0 + P_1)(e_1 + P_0)}}
\]

\[
\frac{(e_1 + P_1)^2}{\rho_1^2} - \frac{(e_0 + P_0)^2}{\rho_0^2} = (P_1 - P_0) \left[ \frac{(e_0 + P_0)}{\rho_0^2} + \frac{(e_1 + P_1)}{\rho_1^2} \right] \]  

(Hugoniot)

\[
e_j = \rho_j c^2 + \frac{P_j}{\Gamma - 1}; \quad j=0,1. \quad (EOS)
\]

\(P = \text{pressure}; \quad e = \text{energy density}; \quad \rho = \text{mass density}\)

\(u_s = \text{shock wave velocity}; \quad u_p = \text{particle flow velocity}\)

\(c = \text{the speed of light.}\)

*subscripts 0 and 1 denote the domains before and after the shock arrival*
The Hugoniot function

Notation: \( \Pi_L \equiv \frac{I_L}{\rho_0 c^3} \); \( \kappa \equiv \frac{\rho_1}{\rho_0} \); \( \kappa_0 \equiv \frac{\Gamma+1}{\Gamma-1} \); \( \Pi = \frac{P_1}{\rho_0 c^2} \); \( \Pi_0 = \frac{P_0}{\rho_0 c^2} \);

The nonrelativistic Hugoniot:
\[
\Pi = \left( \frac{\kappa \kappa_0 - 1}{\kappa_0 - \kappa} \right) \Pi_0
\]

The relativistic Hugoniot for \( \kappa \equiv \frac{\rho_1}{\rho_0} \geq 1 \):

\[
\begin{cases}
\Pi^2 + B \Pi + C = 0 \\
\Pi = \left( \frac{1}{2} \right) \left( -B \pm \sqrt{B^2 - 4C} \right) \\
B = \frac{\left( \Gamma - 1 \right)^2}{\Gamma} \left( \kappa_0 \kappa - \kappa^2 \right) + \Pi_0 \left( \Gamma - 1 \right) \left( 1 - \kappa^2 \right) \\
C = \frac{\left( \Gamma - 1 \right)^2}{\Gamma} \left( \kappa - \kappa_0 \kappa^2 \right) \Pi_0 - \kappa^2 \Pi_0^2
\end{cases}
\]
The non relativistic shock wave equations

\[ e = \rho c^2 + \rho E, \]

\[ P \ll \rho c^2; \quad \rho E \ll \rho c^2 \text{ and } u/c \ll 1 \]

\[ \Downarrow \]

(i) \( u_p = (P_1 - P_0)^{1/2} \left( \frac{1}{\rho_0} - \frac{1}{\rho_1} \right)^{1/2} \)

(ii) \( u_s = \left( \frac{1}{\rho_0} \right) (P_1 - P_0)^{1/2} \left( \frac{1}{\rho_0} - \frac{1}{\rho_1} \right)^{-1/2} \)

(iii) \( E_1 - E_0 = \left( \frac{1}{2} \right) (P_1 + P_0) \left( \frac{1}{\rho_0} - \frac{1}{\rho_1} \right) \)

(iv) \( E_j = \left( \frac{1}{\Gamma - 1} \right) \left( \frac{P_j}{\rho_j} \right) \) for \( j = 0, 1 \)
4- Shock waves in a foil-target collision
Solution of the Hugoniot equations

\[ u_{p1} = -c \sqrt{I_1}; \quad u_{p2} = -c \left( \beta_f - \sqrt{I_2} \right) \left( 1 - \beta_f \sqrt{I_2} \right)^{-1} \]

\[ u_{s1} = -c \sqrt{J_1}; \quad u_{s2} = -c \left( \beta_f - \sqrt{J_2} \right) \left( 1 - \beta_f \sqrt{J_2} \right)^{-1} \]

\[ I_i = \frac{\Pi_i \left[ \Pi_i + (2/3)(\kappa_i - 1) \right]}{(\Pi_i + 1) \left[ \Pi_i + (2/3)\kappa_i \right]} \quad \text{for} \quad i = 1, 2 \]

\[ J_i = \frac{\Pi_i \left[ (2/3)\kappa_i + \Pi_i \right]}{\left[ (2/3)(\kappa_i - 1) + \Pi_i \right][1+\Pi_i]} \quad \text{for} \quad i = 1, 2 \]
The Continuity Equations

Assume: \( \Gamma_t = \Gamma_f \equiv \Gamma = 5/3; \quad \Pi_{0t} = \Pi_{0f} = 0 \)

\[
P_1 = P_2 \implies (1) \quad \kappa_2^2 - 4\kappa_2 = K\left[ \kappa_1^2 - 4\kappa_1 \right]
\]

\[
\mathbf{u}_{p1} = \mathbf{u}_{p2} \implies (2) \quad -\sqrt{I_1} + \beta_r \sqrt{I_1I_2} = \sqrt{I_2} - \beta_r
\]

\[
I_1 = \frac{\Pi_1 \left[ \Pi_1 + \left( \frac{2}{3} \right) (\kappa_1 - 1) \right]}{(\Pi_1 + 1) \left[ \Pi_1 + \left( \frac{2}{3} \right) \kappa_1 \right]}; \quad \Pi_1 = \left( \frac{4}{15} \right) (\kappa_1^2 - 4\kappa_1)
\]

\[
I_2 = \frac{\Pi_2 \left[ \Pi_2 + \left( \frac{2}{3} \right) (\kappa_2 - 1) \right]}{(\Pi_2 + 1) \left[ \Pi_2 + \left( \frac{2}{3} \right) \kappa_2 \right]}; \quad \Pi_2 = \left( \frac{4}{15} \right) (\kappa_2^2 - 4\kappa_2)
\]
The compressions of the shocked target $\kappa_1$ and the shocked flyer $\kappa_2$ for $\rho_{0t}/\rho_{0f} = K = 1000$. 
The pressures of the dimensionless shocked target $\Pi_1$ and the shocked flyer $\Pi_2$ for $\rho_{0t}/\rho_{0f} = K = 1000$. 
The shock and particle velocities accordingly, $u_s$ and $u_p$, for $\rho_{0t}/\rho_{0f} = K = 1000$. (subscript 1 for target, subscript 2 for flyer)
5- Shock waves in a laser-target collision
Dominance of radiation pressure: The piston model

$$P = \frac{2I_L}{c} \left( \frac{1 - \beta}{1 + \beta} \right)$$

$$\beta \equiv \frac{u_p}{c}$$

Relativistic and non-relativistic shock waves

Hugoniot

\[ \frac{u_p}{c} = \sqrt{\frac{2P}{(\Gamma + 1)\rho_0 c^2}} ; \]

\[ \frac{u_s}{c} = \sqrt{\frac{(\Gamma + 1)P}{2\rho_0 c^2}} \]

\[ \frac{\rho}{\rho_0} \]

\[ \frac{P}{\rho_0 c^2} \]

\[ \text{gamma} = \frac{5}{3} \]

\[ P_0 = 1 \text{ bar} \]

\[ \rho_0 = 1000 \text{ gr/cc} \]
**The Temperature problem**

An Example: $I_L = 2.5 \times 10^{22} \, \text{W/cm}^2; \rho_0 = 0.2 \, \text{g/cm}^3 \Rightarrow \begin{cases} \rho = 0.82 \, \text{g/cm}^3 \\ P = 2 \cdot 10^{13} \, \text{atmospheres} \end{cases}$

\[ P = P_i + P_e + P_r; P_i = n_i k_B T_i; P_e = n_e k_B T_e; P_r = (1/3) a T_r^4; a = \left( \frac{1}{15} \right) \left( \frac{k_B^4}{h^3 c^3} \right) = 7.56 \cdot 10^{-15} \text{[erg/(cm}^3 \text{K}^4\text{)]}. \]

\[ k_B T = 31.6 \text{MeV} \]

\[ T_i = T_e = T; \quad T_r = 0 \Rightarrow \quad k_B T = m_p c^2 \left( \frac{A}{Z+1} \right) \left( \frac{P}{\rho c^2} \right) \]

\[ k_B T \sim 26.2 \text{keV}. \]

\[ \left( \frac{m_p}{k_B} \right) \left( \frac{A}{Z+1} \right) \left( \frac{P}{\rho} \right) > T > \left( \frac{3P}{a} \right)^{1/4} \]
If $k_B T > 1 \text{ MeV}$ is possible then

1) For $k_B T > 1 \text{ MeV}$ we have $e^+e^-$ production
   $\Rightarrow$ New Physics $\Rightarrow$ New calculations required

2) For $k_B T > 150 \text{ MeV}$ we have quark-gluon plasma
   $\Rightarrow$ New Physics $\Rightarrow$ New calculations required
6- Nuclear fusion detonation wave.
we are using a 1D plane detonation wave following Chapman-Jouguet (CJ) – Zeldovich-Landau conditions.

chemical based detonation: the energetic material entering the shock front is compressed and thus its temperature rises.

the material transforms exothermally into gasses releasing energy per unit mass \( Q \) that supports the shock.

Our detonation is analogous to above description by changing the chemical energy \( Q \) to nuclear fusion energy.

For DT fusion (17.6MeV fusion energy per reaction) the 3.52MeV of the \( \alpha \) supports the desired steady state detonation wave.
D [cm/s] = The detonation wave velocity
u [cm/s] = fluid velocity; \(c_s\) [cm/s] = speed of sound

\[ P \left[ \text{erg} / \text{cm}^3 \right] = \text{pressure}, \]
\[ \rho \left[ \text{g} / \text{cm}^3 \right] = \text{density}, \]
\[ Q \left[ \text{erg} / \text{g} \right] = \text{nuclear fusion energy that supports the shock} \]

\[ \left( \frac{\rho}{\rho_0} \right)_{cJ} = \Gamma + 1 \frac{1}{\Gamma}; P_{cJ} = \frac{\rho_0 D^2}{\Gamma + 1} \]
\[ \frac{u}{D} = \frac{1}{\Gamma + 1}; \frac{c_s}{D} = \frac{\Gamma}{\Gamma + 1} \]
\[ \frac{Q}{D^2} = \frac{1}{2(\Gamma^2 - 1)} \]
In our model

\[
Q \left[ \frac{J}{kg} \right] = \left( \frac{E_\alpha}{\rho} \right)^{t=\tau_L} \int_0^1 dt \left( \frac{dn_\alpha}{dt} \right) \frac{1}{2} (1 + f_\alpha)
\]

\(f_\alpha\) is given in
S. Yu. Guskov and V. Rozanov
Nuclear fusion by Inertial Confinement
pp293-320, CRC press
Eds. G. Velarde, Y. Ronen and J. M. Martinez Val

\[
f_\alpha = \begin{cases} 
  \frac{3}{2} x_\alpha - \frac{4}{5} x_\alpha^2 & x_\alpha < \frac{1}{2} \\
  1 - \frac{1}{4 x_\alpha} + \frac{1}{160 x_\alpha^3} & x_\alpha \geq \frac{1}{2}
\end{cases}
\]

\[
x_\alpha (\tau) = \frac{R}{R_\alpha}; R = (u_s - u_p) \tau_L
\]

\[
R_\alpha [cm] = \frac{1}{\kappa \rho_0} \left[ \frac{1.5 \times 10^{-2} T_e (keV)^{5/4}}{1 + 8.2 \times 10^{-3} T_e (keV)^{5/4}} \right]
\]
The fusion energy $Q$ per unit mass released in the forward shock wave direction as a function of time for a detonation with $\Gamma = 3$.

(the laser: $I_L = 1.75 \times 10^{23} \text{W/cm}^2/\tau_L = 1$; the target: pre-compressed to 900 g/cm$^3$.)
7. A Novel fast ignition scheme
Inertial Confinement Fusion (ICF)

Spark ignition ICF: NIF

Fast ignition ICF

J. Nuckolls et al., Nature 239, 139 (1972)


Fast ignition needs less laser energy (~300 kJ) than central spark scheme (~3MJ).

Therefore

Fast ignition has the potential to be the best route to achieve nuclear fusion as an energy source.

But there are many schemes of fast ignition:

Ignition by laser shock wave

Many ns lasers

CH

0.1 mm $\sim \Delta R_0$

DT solid (0.2 g/cm$^3$)

1 mm $\sim R_0$

DT gas (0.3 mg/cm$^3$)

ps laser

$r=67 \mu m$

$2R_L=7.2 \mu m$

0.72 $\mu m$
Ignition by laser accelerated foil impact

The micro-foil

ps laser

cone of gold

ns laser

ns laser

CH

DT gas

DT solid
DT ignition

\[ \Pi_L = 6.65 \times 10^{-5}; \quad \tau_L = 1 \text{ps}; \quad \rho_0 = 250 \left[ \frac{g}{cm^3} \right] \Rightarrow \]

\[ \rho = \kappa \rho_0 = 10^3 \left[ \frac{g}{cm^3} \right] \]

\[ I_L = \rho_0 c^3 \Pi_L = 4.5 \times 10^{22} \left[ \frac{W}{cm^2} \right] \]

\[ \ell_s = (u_s - u_p) \tau_L \approx 1 \mu m; \quad S = \pi R_L^2 = \pi (1.5 \ell_s)^2 = 7.04 \times 10^{-8} \left[ cm^2 \right] \]

\[ W_L [J] = I_L S \tau_L = 3.2 \text{kJ} \]
$T_e$ and $T_i$ as a function of time for DT satisfying the ignition criterion
$T_e$ and $T_i$ as a function of time for $pB^{11}$ satisfying the ignition criterion
8- The 2-temperatures ignition criteria for DT and pB$^{11}$
\[ A_1 + A_2 \rightarrow A_3 + A_4 + E_f \]
\[ E_f = E_\alpha + E_{\text{others}} \]
\[ W_f \left[ \frac{\text{erg}}{\text{cm}^3 \cdot \text{s}} \right] = n_1 n_2 < \sigma v >_{12} E_\alpha \]

\[ W_f - \sum W(\text{losses}) \geq 0 \]
\[ f_\alpha W_f - W_B - W_{he} - W_m \geq 0 \]
Two temperature equations

\[
\left( \frac{3}{2} \right) \frac{d}{dt} (n_e k_B T_e) = \eta_d W_d + W_{ie} - W_B + f_\alpha \eta_f W_f
\]

\[
\left( \frac{3}{2} \right) \frac{d}{dt} (n_i k_B T_i) = (1 - \eta_d) W_d - W_{ie} + f_\alpha (1 - \eta_f) W_f
\]

For DT fusion: \[\frac{dn_D}{dt} = \frac{dn_T}{dt} = -\frac{dn_\alpha}{dt} = -n_D n_T < \sigma v >_{DT}\]

For pB\textsuperscript{11} fusion: \[\frac{dn_p}{dt} = \frac{dn_B}{dt} = -\left( \frac{1}{3} \right) \frac{dn_\alpha}{dt} = -n_p n_B < \sigma v >_{pB}\]
$D + T \rightarrow n + \alpha + 17,589 \text{ keV}$

\[ n_e \left[ \text{cm}^{-3} \right] = n_i \left[ \text{cm}^{-3} \right] = \left( \frac{\rho}{2.5m_p} \right) = 2.39 \times 10^{23} \rho \]

\[ W_{f,DT} \left[ \frac{\text{erg}}{\text{cm}^3 \cdot s} \right] = 8.07 \times 10^{40} < \sigma v >_{DT} \rho^2 \]

\[ W_B \left[ \frac{\text{erg}}{\text{cm}^3 \cdot s} \right] = 8.58 \times 10^{21} \rho^2 T_e (eV)^{0.5} \left( 1 + \frac{2T_e (eV)}{0.511 \times 10^6} \right) \]

\[ W_m \left[ \frac{\text{erg}}{\text{cm}^3 \cdot s} \right] = 1.02 \times 10^{18} \left[ T_e (eV) + T_i (eV) \right]^{1.5} \left( \frac{\rho}{R} \right) \]

\[ W_{he} \left[ \frac{\text{erg}}{\text{cm}^3 \cdot s} \right] = \frac{3.11 \times 10^9 T_e (eV)^{7/2}}{R^2 \ln \Lambda} \]

Ignition requirement: \( a(T_e, T_i) (\rho \cdot R)^2 + b(T_e, T_i) (\rho \cdot R) + c(T_e) \geq 0 \)
Equal $\rho R$ as a function of ions and electrons temperatures for DT ignition

S. Eliezer et al., Laser and Particle Beams 33, 577-589 (2015).
Equal $\rho R$ as a function of ions and electrons temperatures for $pB^{11}$ ignition for $\varepsilon = n_B/n_p = 1/3$

\[ p + ^{11}B \rightarrow 3\alpha + 8,700 \text{ keV} \]

9- Summary and Conclusions
NOVEL PROPOSALS:

The Recent and future developments of high power lasers in the multi Petawatt domain

1- Relativistic Shock Waves in the laboratory (P ~ 10^{15} atmospheres)
2- New scheme for Fast ignition by a shock wave
3- Micro-accelerators to relativistic velocities (colliding foils)
4- Quark-gluon plasma?
Solving the Energy Problem
Mark Twain (1835-1910)
“And what is a man without energy? Nothing-nothing at all”

ICF solution:
1. Understanding the Physics
2. Energy conservation
3. Economically practical
   Gain=Output energy/laser input energy > 100
   Cost per target ~ 0.10 US $
4. Simplicity
   ~10^8 laser shots per year
What I learned from Noah's Ark?
The Bible: book of GENESIS

ECLIM 2006 (Madrid)

• One: Don't miss the boat

• Two: Remember that we are all in the same boat

• Three: Plan ahead. It wasn't raining when Noah built the Ark

• Four: Stay fit. When you're 600 years old, someone may ask you to do something really big

• Five: Don't listen to critics; just get on with the job that needs to be done

• Remember, the Ark was built by amateurs; the Titanic by professionals
THANK YOU