New Features
in
COMPLEX INTERFEROMETRY Diagnostics

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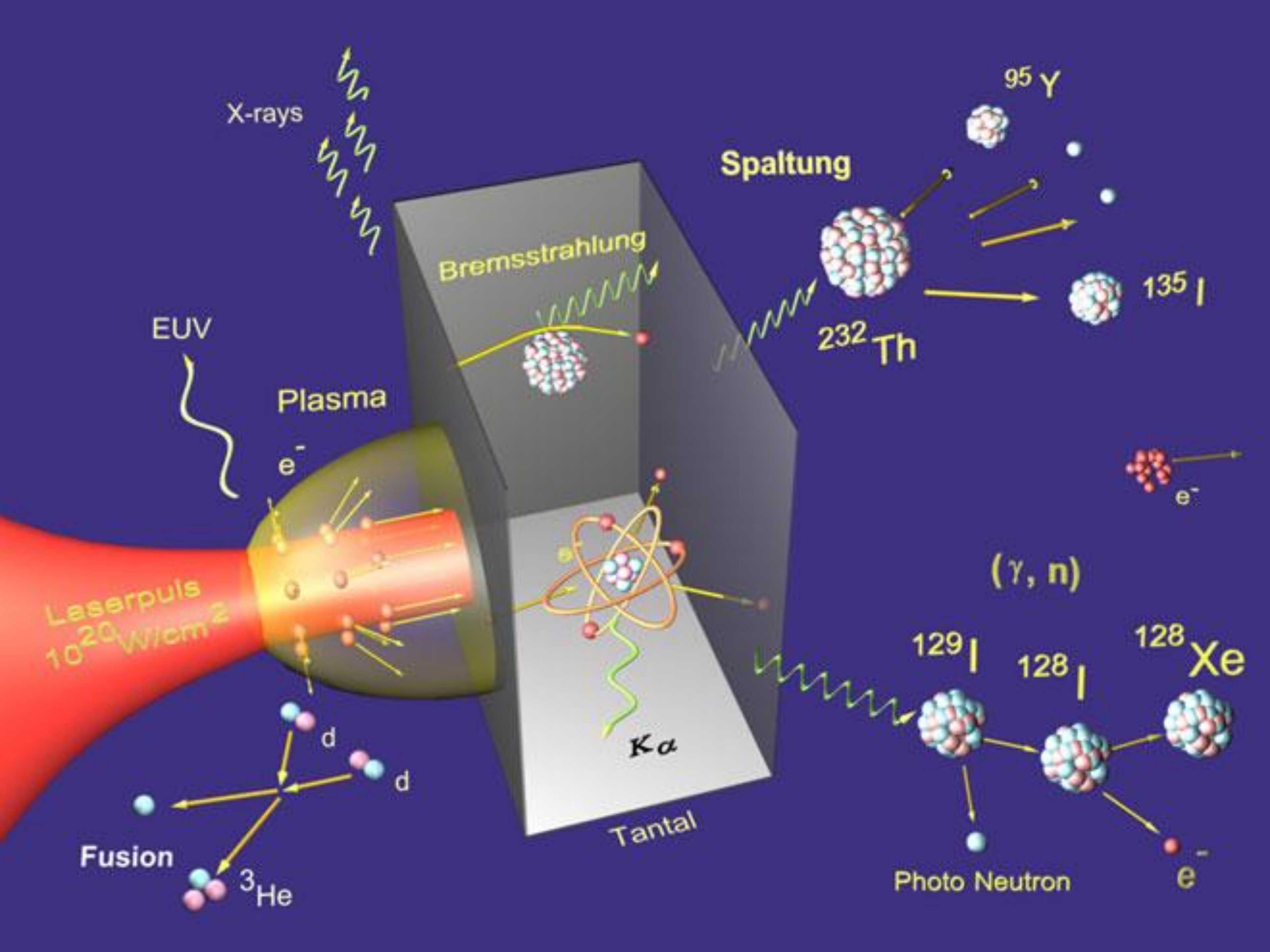
HOW *COMPLEX INTERFEROMETRY* CAME TO ITS EXISTENCE ?

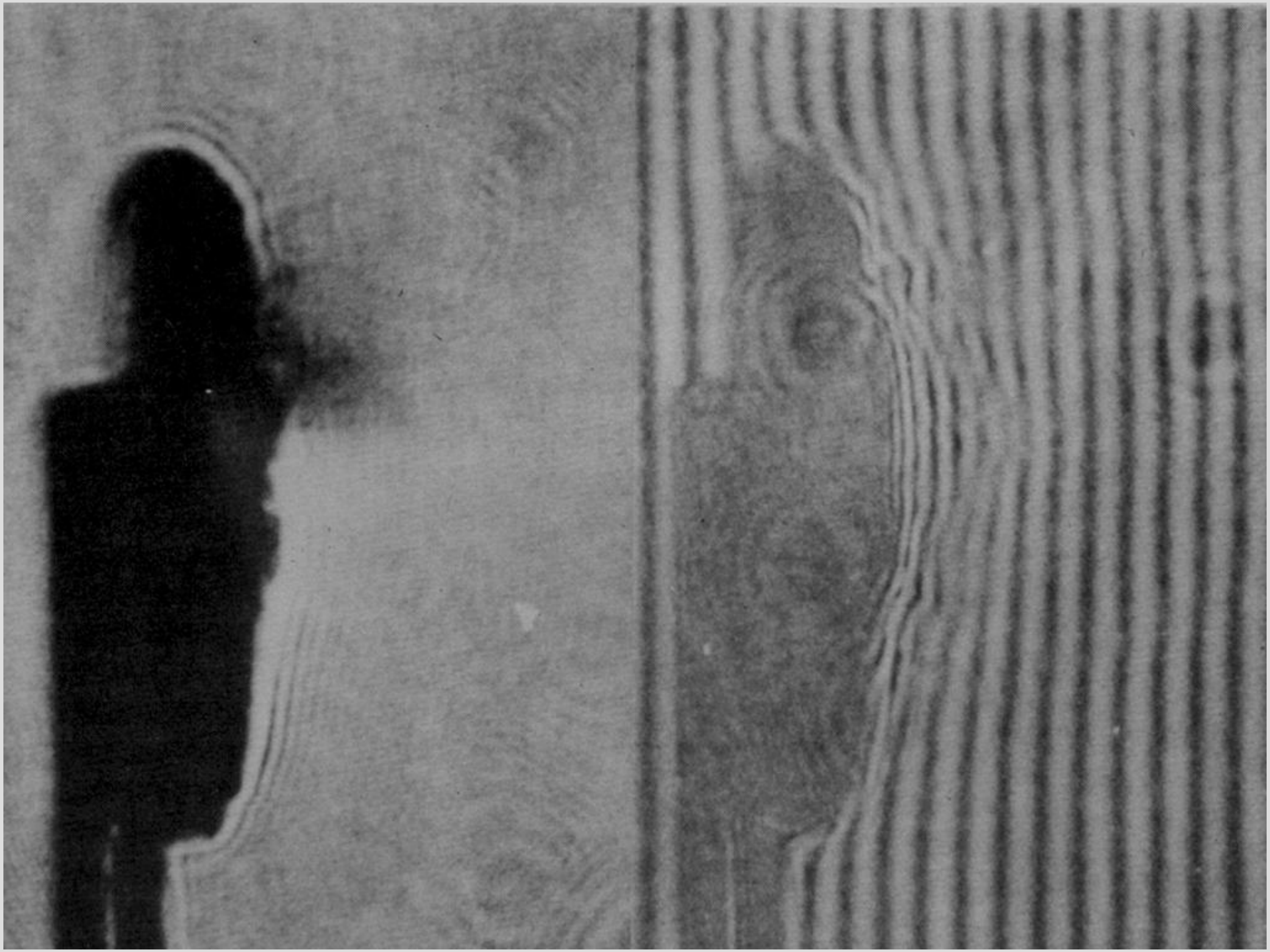
Through measurements of magnetic field profiles spontaneously generated in laser produced plasmas

$$\theta = 1.51\, \lambda_p^2 \int \frac{nB\,dl}{10^{23}(1 - n/n_{cp})^{1/2}}$$

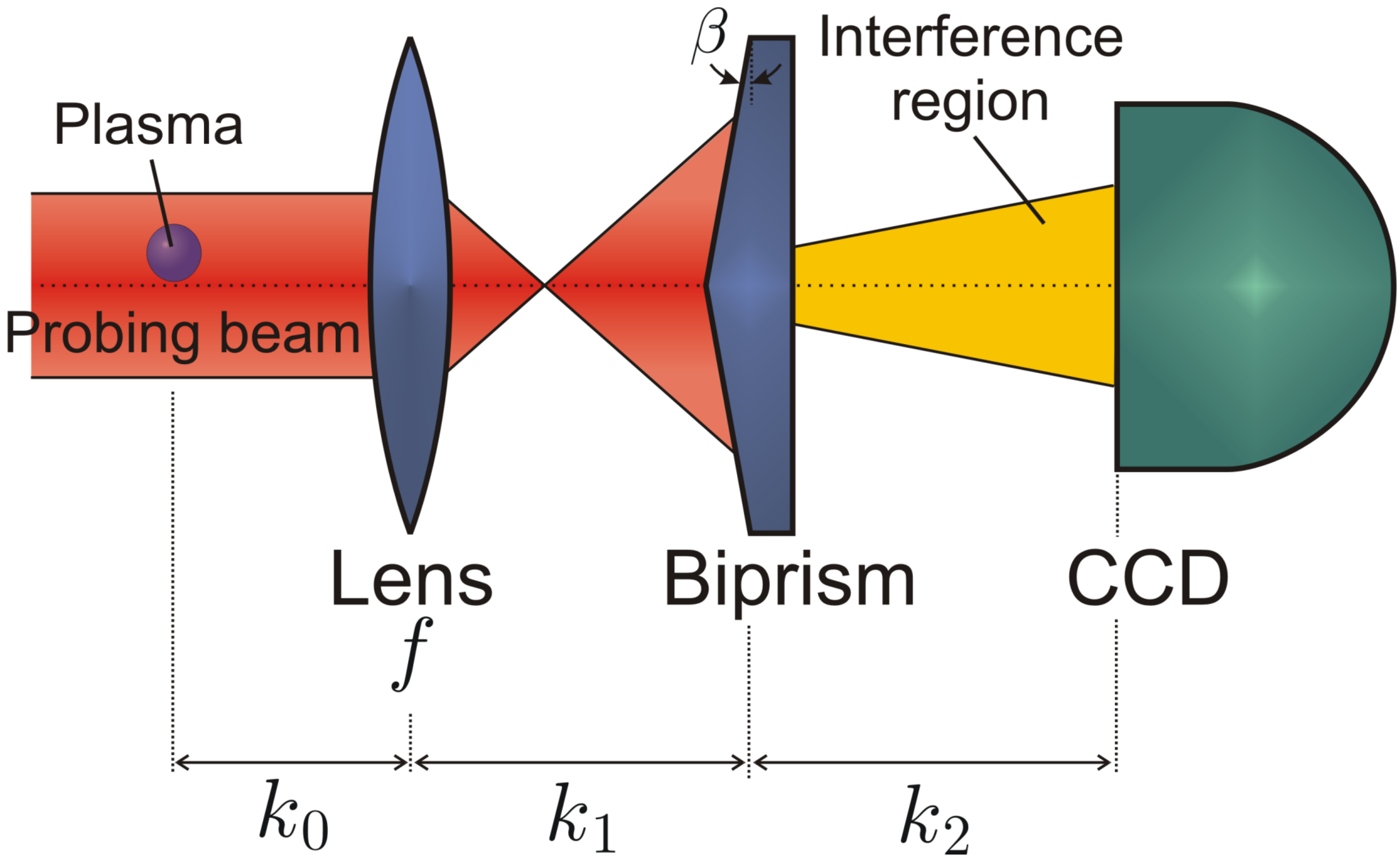


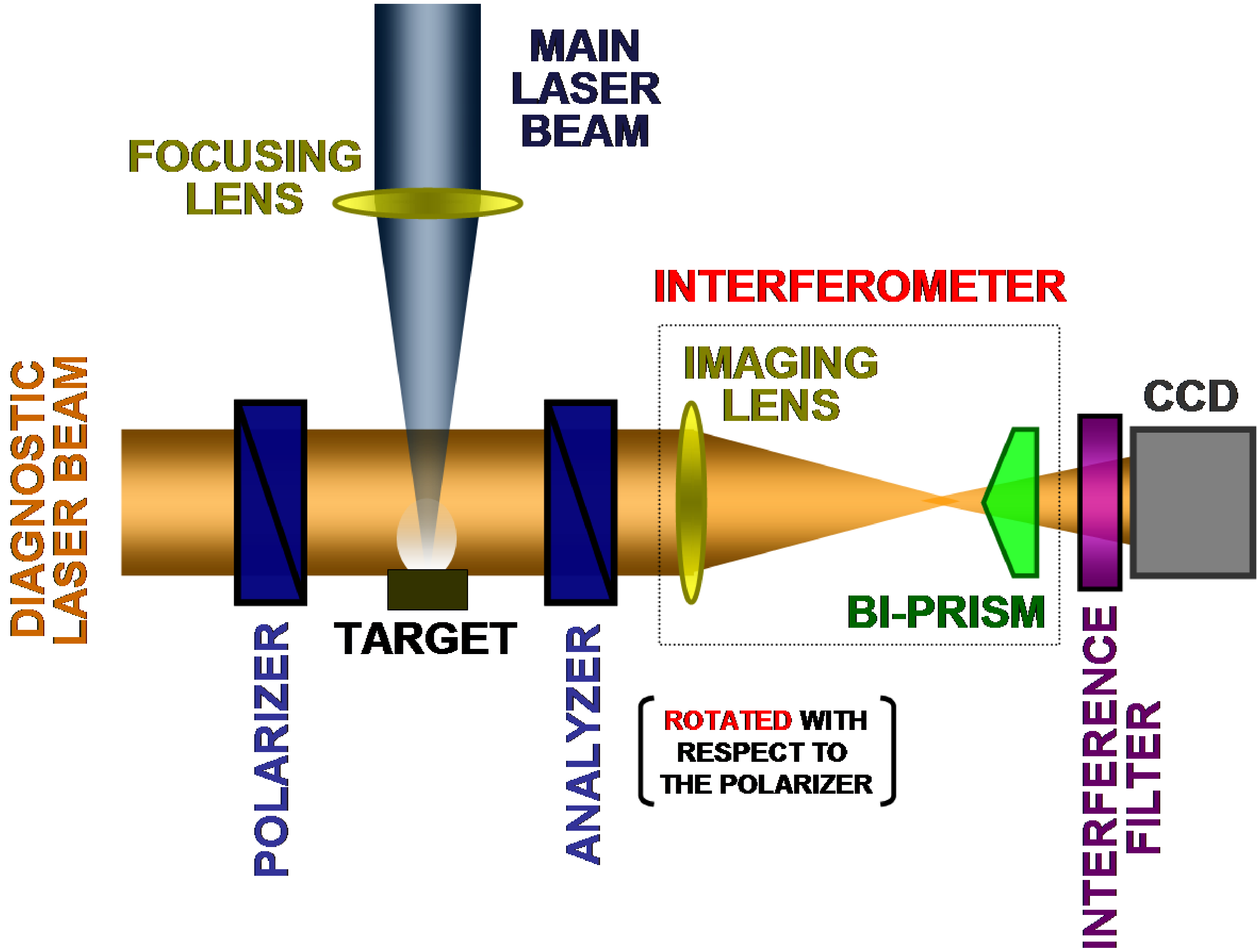
X-rays
95 Y
Spaltung
Bremsstrahlung
135 I
232 Th
EUV
Plasma
e-
e-
(γ, n)
Laserpuls
10^20 W/cm^2
129 I
128 I
128 Xe
d
d
Kα
Tantal
Fusion
3He
Photo Neutron
e-

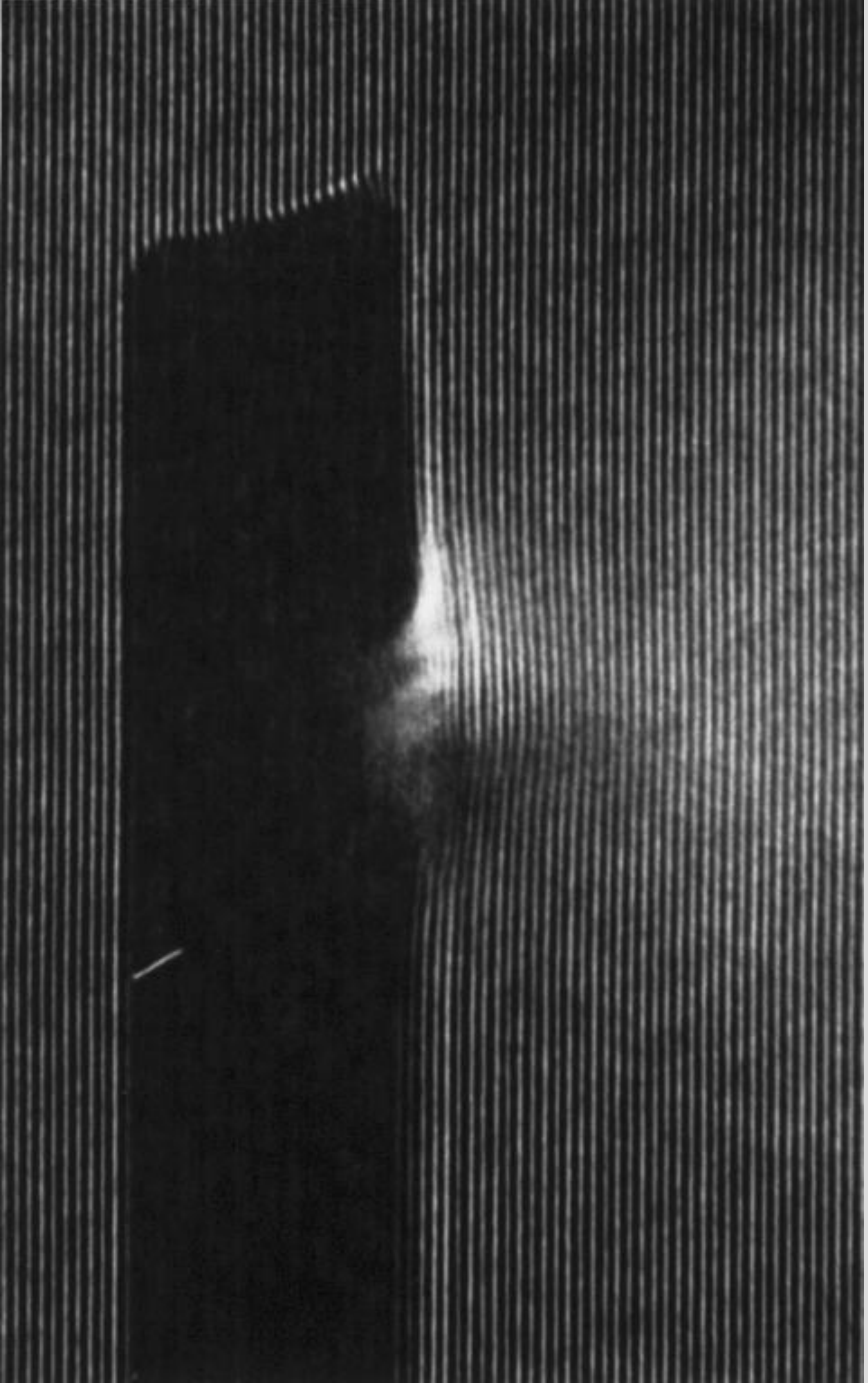
Spontaneous magnetic fields in laser produced plasma



Nomarski type Interferometer with Fresnel Biprism




MAIN LASER BEAM
FOCUSING LENS
INTERFEROMETER
IMAGING LENS
CCD
DIAGNOSTIC LASER BEAM
BI-PRISM
POLARIZER
TARGET
ANALYZER
ROTATED WITH RESPECT TO THE POLARIZER
INTERFERENCE FILTER



The very first
Complex Interferogram

Milan Kalal
Barry Luther-Davies

1986

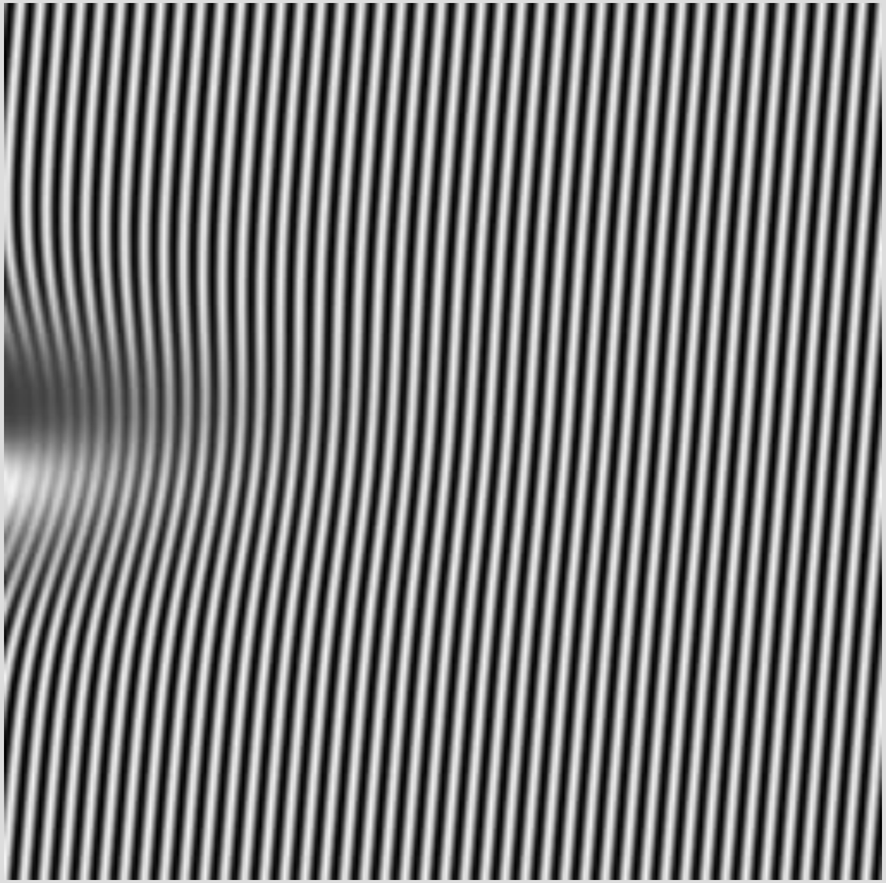
Australian National University
Canberra, Australia

Measurement of
Spontaneously Generated
Magnetic Fields
In Laser Produced Plasma

Complex Interferogram

Synthetic

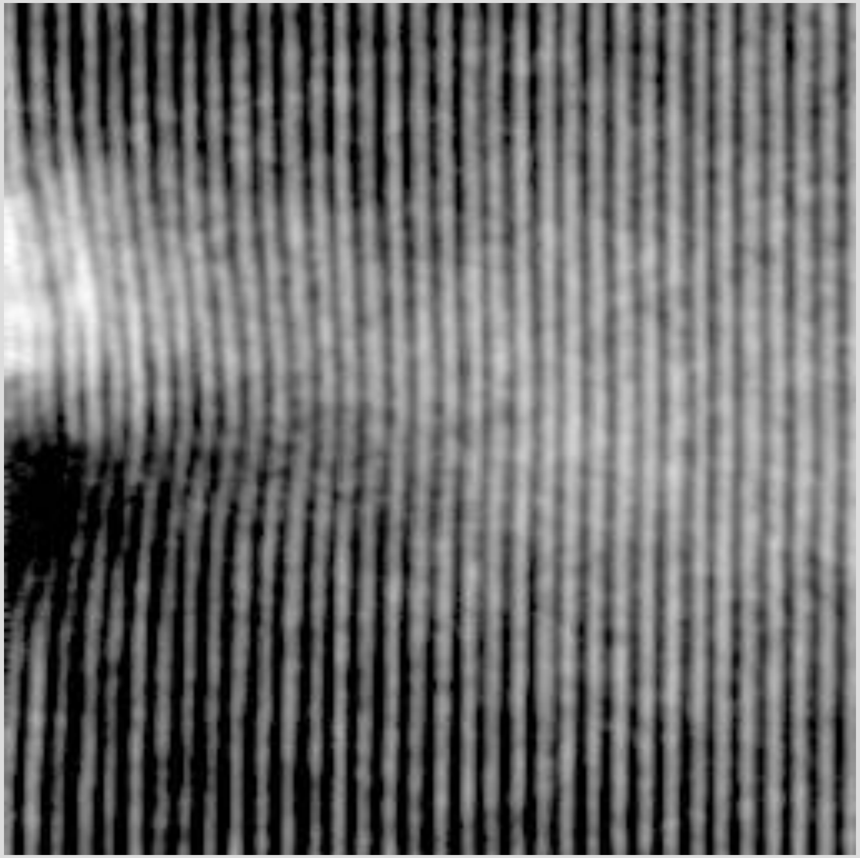
3 Degrees of Freedom



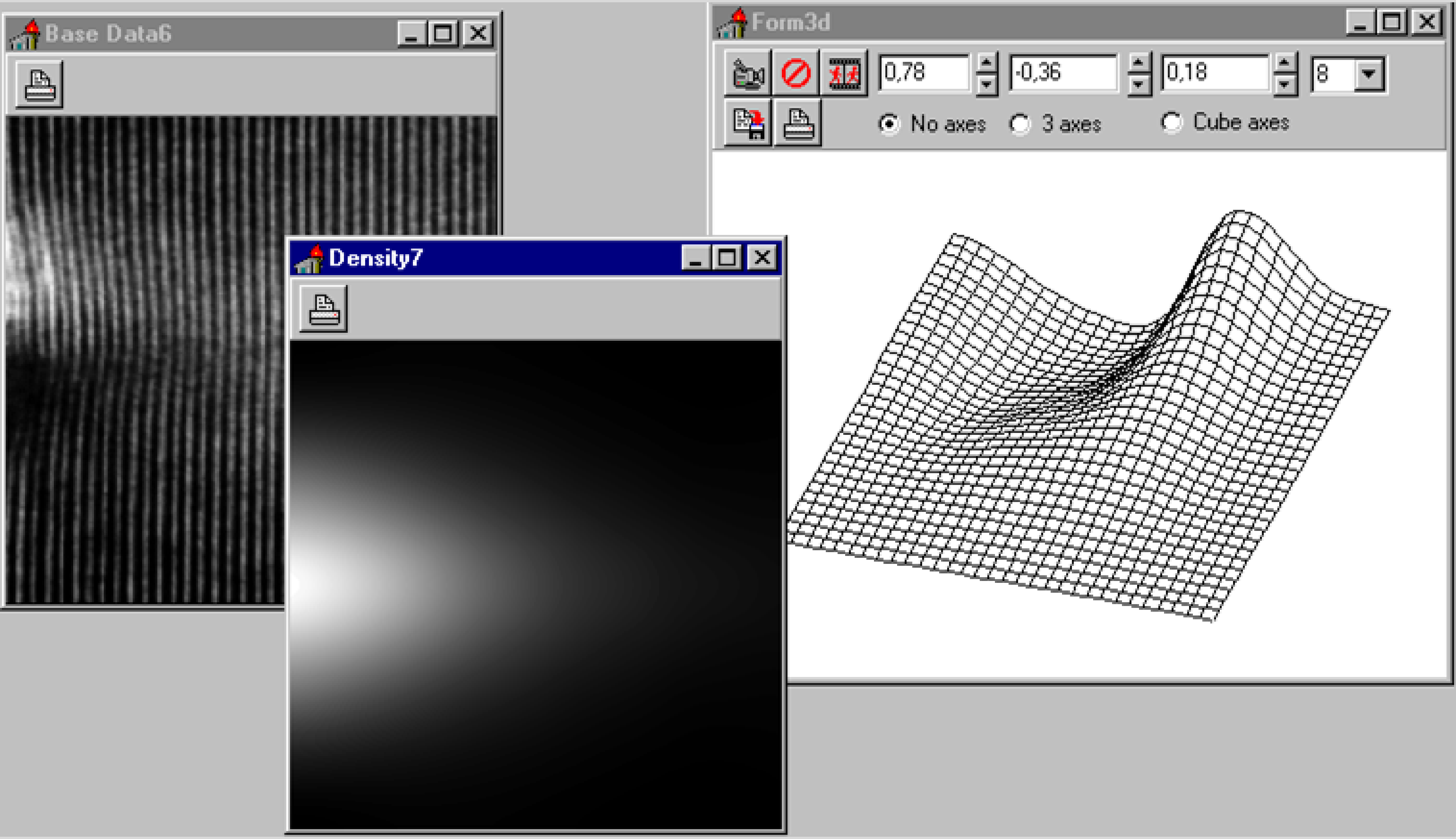
Complex Interferogram

Experimental

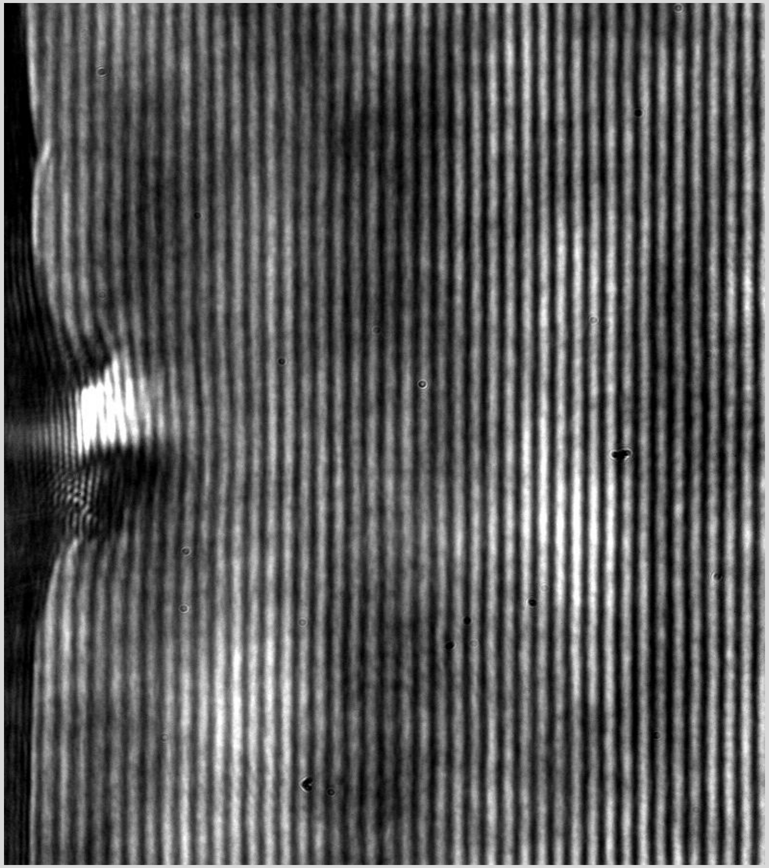
3 Degrees of Freedom



EXAMPLE OF THE VERY FIRST COMPLEX INTERFEROGRAM ANALYSIS



The very first *Complex Interferogram* of spontaneously generated MG fields successfully recorded at PALS (2015)



T. Pisarczyk, M. Kalal *et al*, Physics of Plasma 22, 102706 (2015)

Space-time resolved measurements of spontaneous magnetic fields in laser-produced plasma

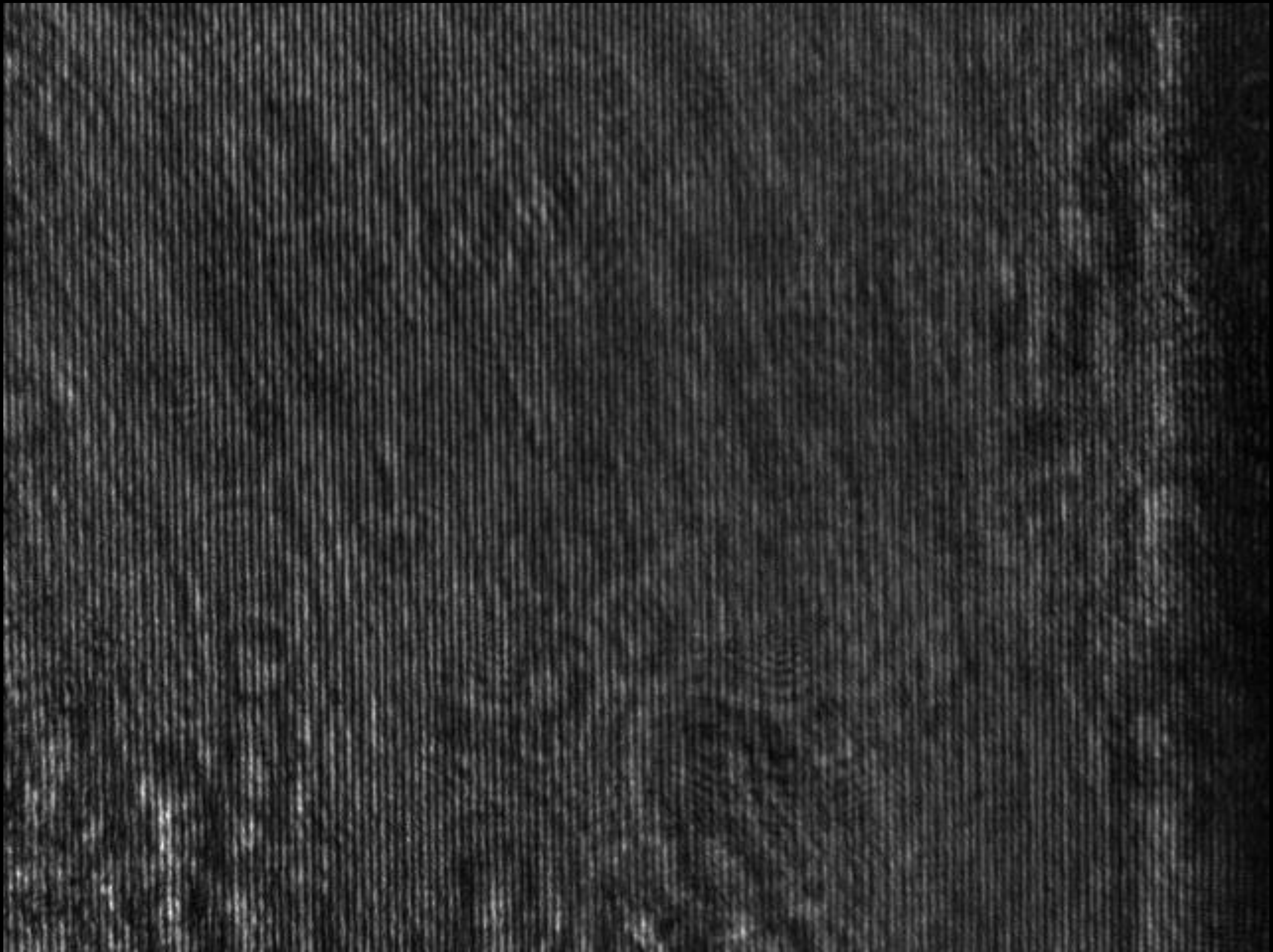
KEY REQUIREMENT

!!!!!

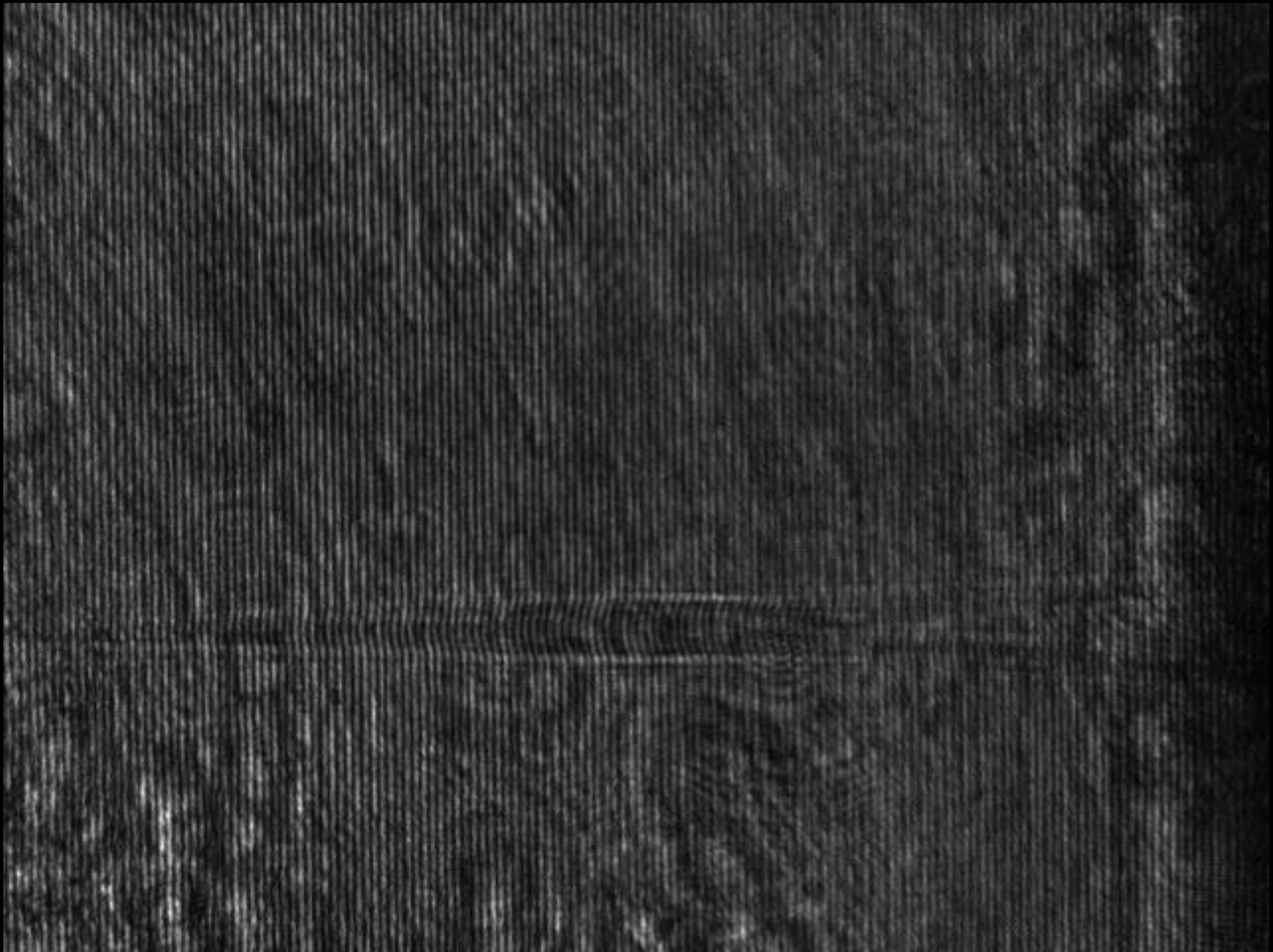
STABILITY

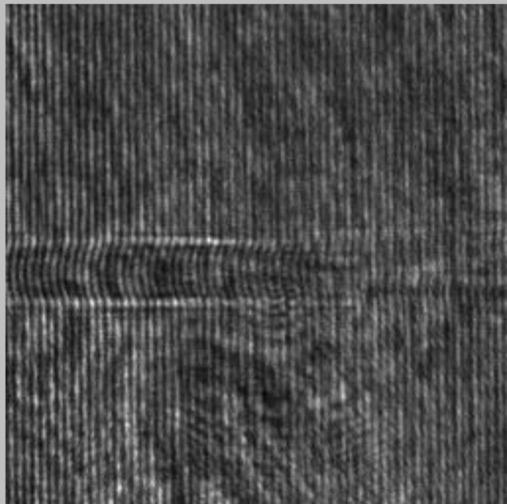
OF THE DIAGNOSTIC SYSTEM

Interferometry, HILT, Szeged, Hungary

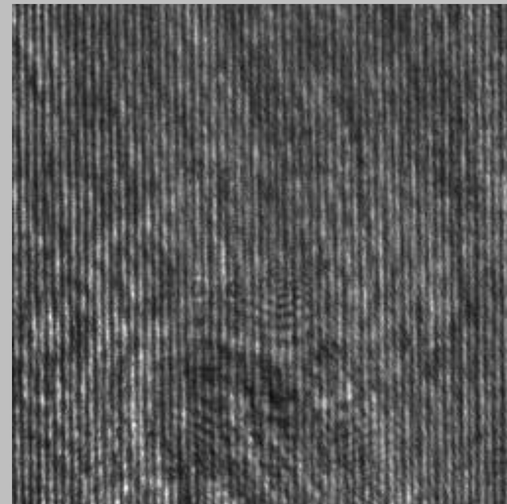


Interferometry, HILT, Szeged, Hungary

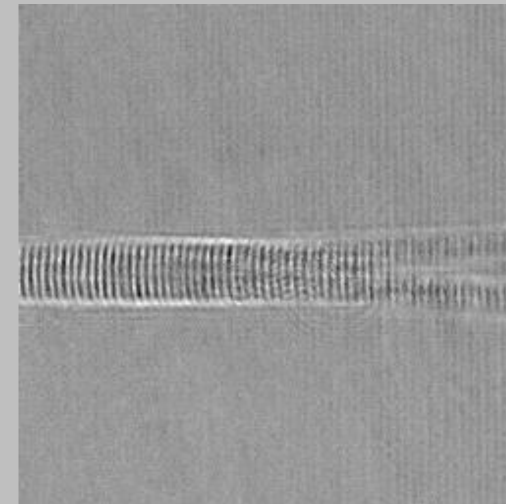




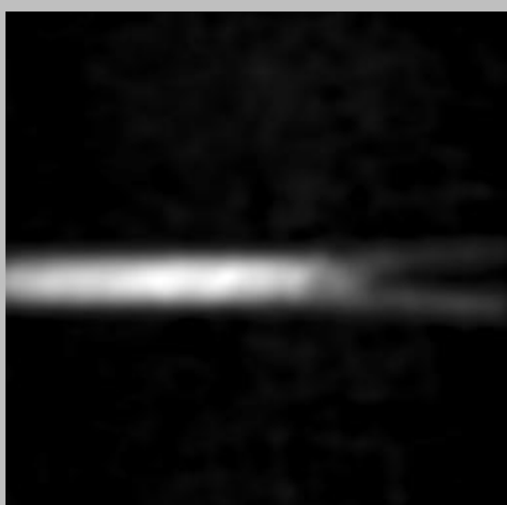
Signal



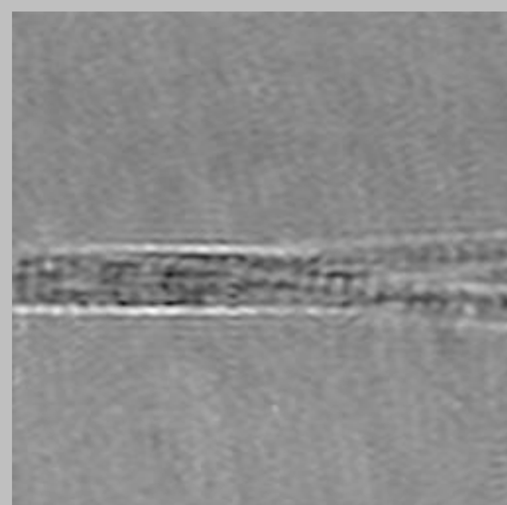
Reference



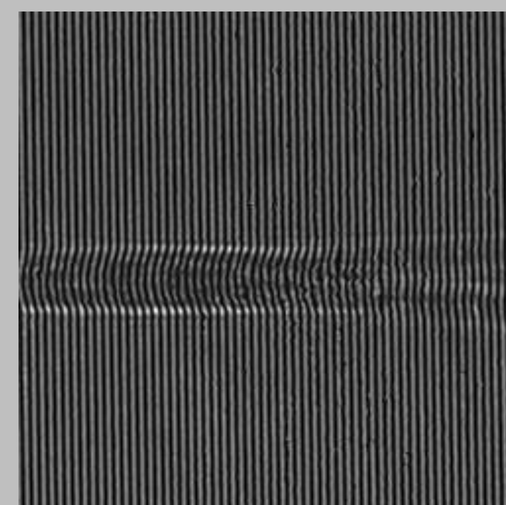
Difference



Phase shift



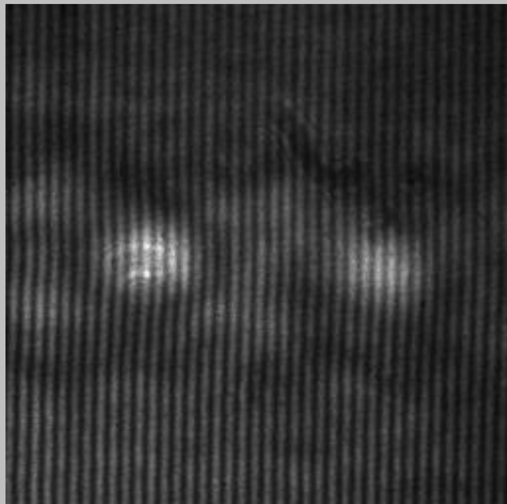
Amplitude



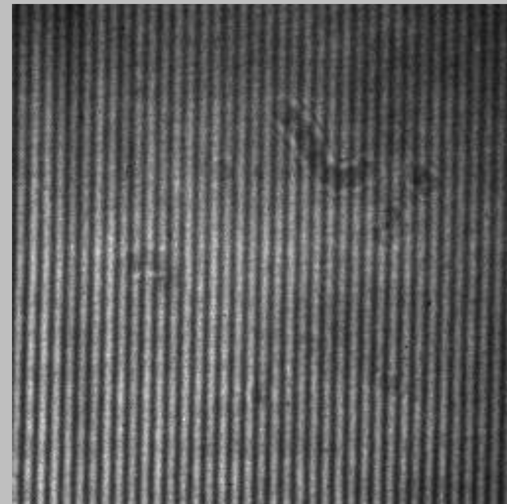
Synthetic

National Commision of Atomic Energy, Buenos Aires, Argentina

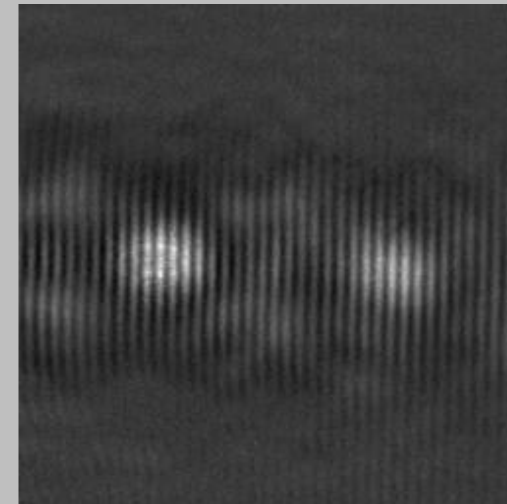
National Commision of Atomic Energy, Buenos Aires, Argentina



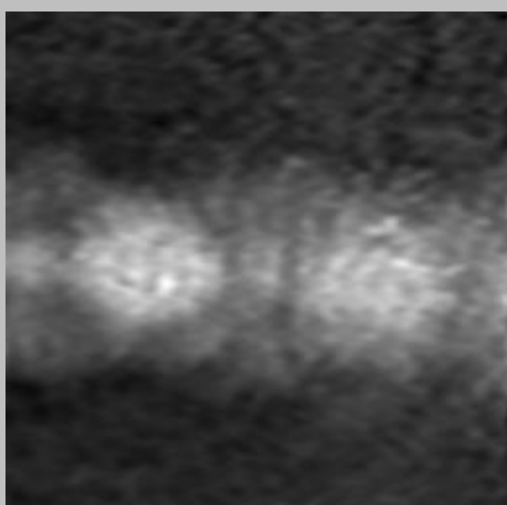
Signal



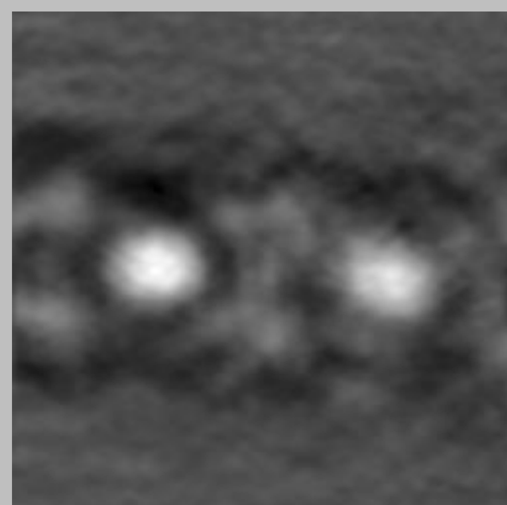
Reference



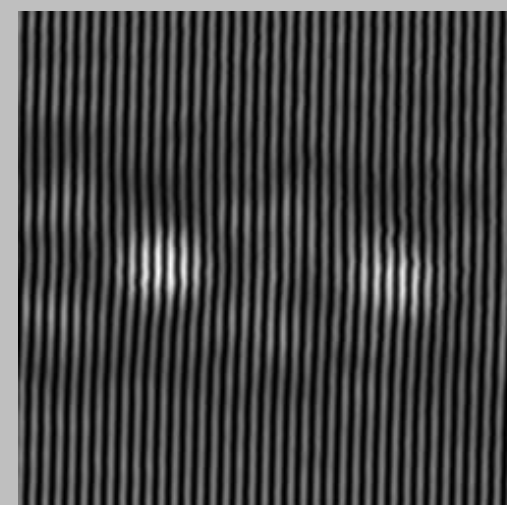
Difference



Phase shift



Amplitude



Synthetic

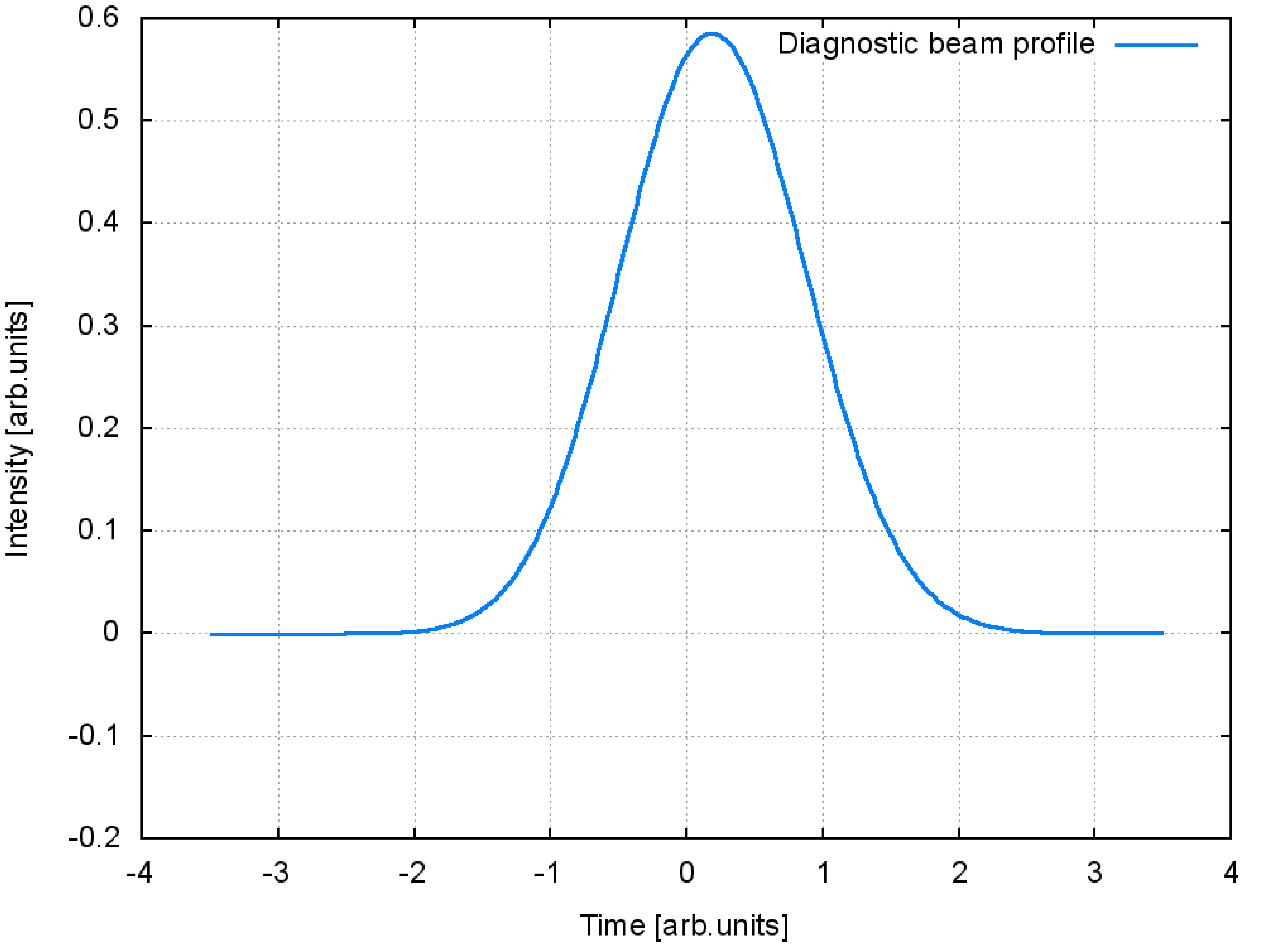
COMPLEX INTERFEROGRAM ANALYSIS

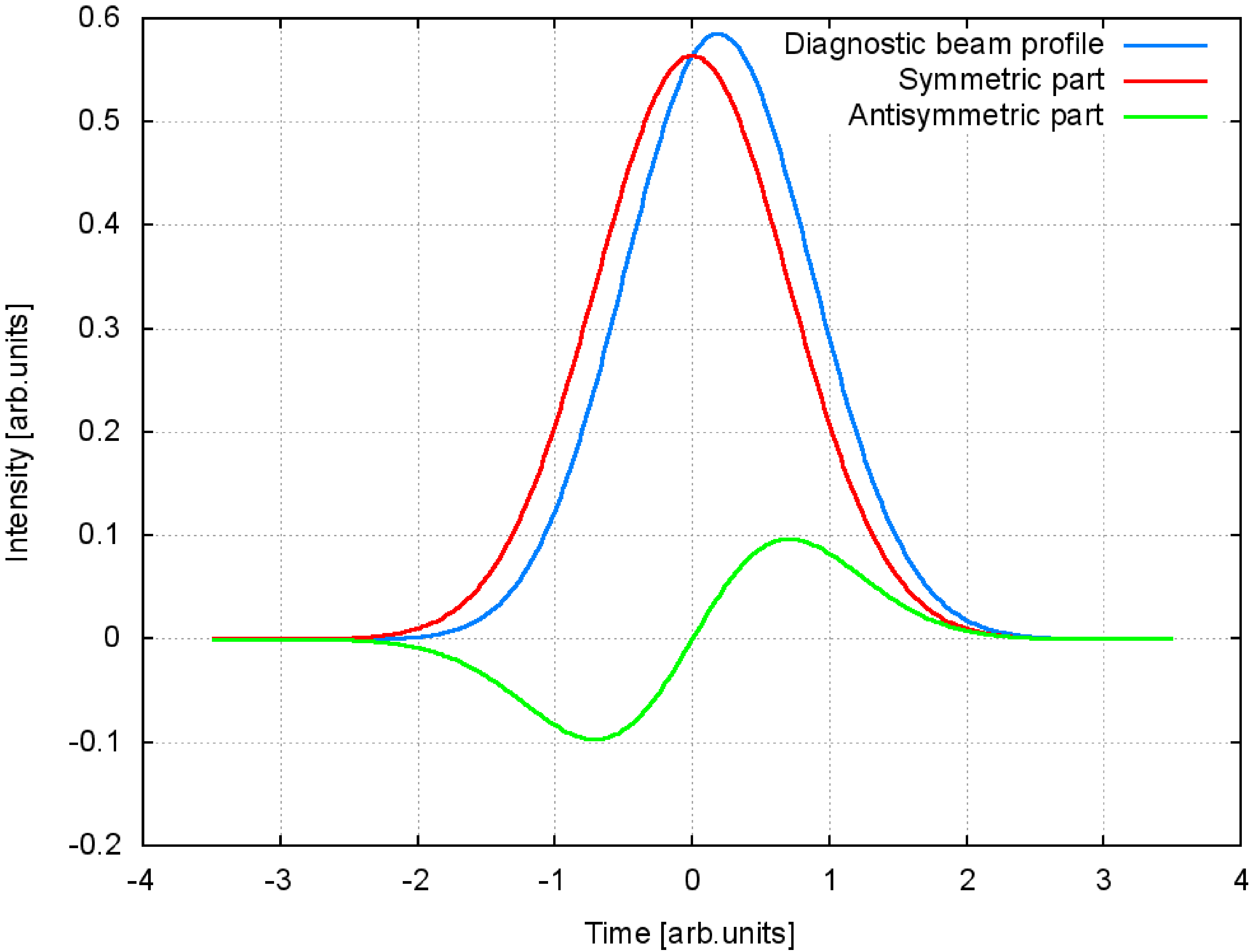
While making interferometry, the ***final interferogram i(y,z)*** is a ***superposition*** of a series of ***instantaneous interferograms i(y,z,t)*** recorded trough the ***duration*** of the ***diagnostic*** beam pulse ***f(t)***

$$i(y,z,t) = a_r^2(y,z,t) f(t) + a_s^2(y,z,t) f(t) +$$
$$+2a_r(y,z,t)a_s(y,z,t)\cos\left[2\pi(\omega_0 y + \nu_0 z) + \varphi(y,z,t)\right] f(t)$$

Here ***$a_r(y,z,t)$*** and ***$a_s(y,z,t)$*** are the ***instantaneous amplitudes*** of the ***reference*** and the ***signal*** beams, ***ω_0*** and ***ν_0*** are the ***spatial frequencies*** in the directions ***y*** and ***z*** (in the ***plane of interferogram***), respectively,

and ***$\varphi(y,z,t)$*** is the ***instantaneous phase shift*** between the ***reference*** and the ***signal*** beam.


Diagnostic beam profile
Intensity [arb.units]
Time [arb.units]
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-4
-3
-2
-1
0
1
2
3
4


Diagnostic beam profile
Symmetric part
Antisymmetric part
Intensity [arb.units]
Time [arb.units]
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-4
-3
-2
-1
0
1
2
3
4

The shape of the ***diagnostic*** pulse ***f(t)*** can be defined (without any lose of generality) to satisfy the following criteria

$$f(t) = f_s(t) + f_a(t) \geq 0$$

Intensity cannot be ***negative***

Time ***t=0*** is selected to be in the ***center*** of its ***symmetric*** - ***$f_s(t)$*** as well as ***antisymmetric*** - ***$f_a(t)$*** part.

As a result of that the following expressions will be true:

$$\int_{-\infty}^{+\infty} t^{2n+1} f_s(t)\,dt = 0$$

and

$$\int_{-\infty}^{+\infty} t^{2n} f_a(t)\,dt = 0$$

$$\int_{-\infty}^{+\infty} f(t)\,dt = \int_{-\infty}^{+\infty} f_s(t)\,dt = 1$$

Intensity can be ***normalized***

Let us now suppose that, in principle, both the ***phase shift*** ***$\varphi(y,z,t)$*** and the ***amplitudes*** ***$a_s(y,z,t)$*** and ***$a_r(y,z,t)$*** of the ***diagnostic*** beam can ***evolve*** in time due to ***temporal changes*** of characteristics of the ***object*** under investigation as well as of its ***own***.

Keeping this in mind it becomes useful to express these quantities in the form of the ***first order Taylor expansion*** with ***representative*** values ***$\varphi(y,z)$***, ***$a_s(y,z)$*** and ***$a_r(y,z)$*** as well as the corresponding ***time derivatives*** taken at the time ***$t=0$***

$$\varphi(y,z,t) = \varphi(y,z) + \varphi'(y,z)t$$

$$a_s(y,z,t) = a_s(y,z) + a_s'(y,z)t$$

$$a_r(y,z,t) = a_r(y,z) + a_r'(y,z)t$$

Performing the ***time integration*** (substituting the expansions)

$$i(y,z)=\int_{-\infty}^{+\infty} i(y,z,t)dt$$

and considering only the ***most relevant terms*** we can get the ***modified*** form of the ***usual*** interferogram formula

$$i(y,z)=a_r^2(y,z)+a_s^2(y,z)+$$
$$+2a_r(y,z)a_s(y,z)\left|q(y,z)\right|\cos\left[2\pi(\omega_0 y+\nu_0 z)+\varphi_{total}(y,z)\right]$$

Please, note the presence of the function ***|q(y,z)|*** !!!

As well as the ***total phase shift*** $\varphi_{total}(y,z)$!!!

Their meaning will be explained in the following slides.

$$\varphi_{total}(y,z) = \varphi_p(y,z) + \varphi_{derr}(y,z) + \varphi_{serr}(y,z)$$

In this expression for the ***total phase shift*** the meaning of the ***individual*** **contributions** is the following:

$\varphi_p(y,z)$ stands for the ***pure*** phase shift caused by the ***object itself*** (this is the function ***we are looking for*** !!!)

$\varphi_{derr}(y,z)$ stands for the ***error*** caused by the ***diagnostic system*** itself !!! (***interferometer setup*** and the diagnostic beam ***wave front quality*** !!!)

$\varphi_{serr}(y,z)$ stands for the ***systematic error*** caused by the degree of ***asymmetry*** of the ***diagnostic pulse f(t)***

The ***q(y,z)*** function comes from the following time integration

$$q(y,z)=\int_{-\infty}^{+\infty}\exp[i\varphi'(y,z)t]\,f(t)\,dt$$

In the case of the ***symmetric*** diagnostic beam profile ***f(t)*** this expression ***simplifies*** to the form

$$q(y,z)=\int_{-\infty}^{+\infty}\cos[\varphi'(y,z)t]\,f(t)\,dt$$

the ***q(y,z)*** function becomes the function with ***real*** values

$$0<q(y,z)\leq 1$$

For typical ***symmetric*** diagnostic beam profiles ***f(t)*** this function is ***monotonically decreasing*** function of ***φ'(y,z)***. This makes finding the ***inversion*** process possible, provided the exact time profile of ***f(t)*** is available – either ***analytically*** or ***numerically*** (by ***sampling*** the ***f(t)*** time profile).

Gaussian pulse can be used as a ***typical example*** of the ***symmetric*** diagnostic beam with ***analytical*** profile ***f(t)*** to illustrate this topic (already normalized to unity):

$$f(t)=\frac{1}{\sqrt{\pi}\tau}\exp\left(-\frac{t^2}{\tau^2}\right)$$

After its substitution in the integral expression for the ***q(y,z)*** function it is possible to find the solution ***φ'(y,z)*** in the form

$$\varphi'(y,z)=\frac{2}{\tau}\sqrt{-\ln q(y,z)}$$

In the case of the ***symmetric*** diagnostic beam profile ***f(t) no*** systematic error $\varphi_{serr}(y,z)$ will be generated.

However, in more ***practical*** cases (with some degree of ***asymmetry*** of the diagnostic beam profile ***f(t)***), it would be very convenient to be able to make an ***estimate*** of the value of this systematic error $\varphi_{serr}(y,z)$.

Such estimate is directly related to the fact that the $\varphi'(y,z)$ function can be reconstructed reasonably well even in the case of ***f(t) asymmetry***. And the ***imaginary*** part of ***q(y,z)*** comes from the following integration:

$$q_i(y,z)=\int_{-\infty}^{+\infty}\sin[\varphi'(y,z)t]\,f_a(t)dt$$

Therefore

$$|q_i(y,z)| \ll q_r(y,z)$$

When it was taken into account that both ***antisymmetric*** functions $f_a(t)$ and $\sin[\varphi'(y,z)t]$ go through ***zero*** values at the time $t = 0$ (unlike $f_s(t)$ and $\cos[\varphi'(y,z)t]$).

Therefore, the following approach can be employed:

First of all the reconstructed values $|q(y,z)|$ will be considered as a reasonable approximation for the ***real*** part $q_r(y,z)$.
This will provide the way of reconstructing the $\varphi'(y,z)$.
Subsequently, the ***imaginary*** part $q_i(y,z)$ will be calculated as described in the previous slide.

Finally, the $\varphi_{serr}(y,z)$ can be determined

$$\varphi_{serr}(y,z) = \arcsin \frac{q_i(y,z)}{|q(y,z)|}$$

and ***subtracted*** from the reconstructed phase shift $\varphi_{total}(y,z)$.

Using the formula $\cos x = (e^{ix} + e^{-ix})/2$

the expression for an ***interferogram*** takes the form

$$i(y,z) = b(y,z) + v(y,z)\exp[2\pi i(\omega_0 y + \upsilon_0 z)] + $$
$$+v^*(y,z)\exp[-2\pi i(\omega_0 y + \upsilon_0 z)]$$

where

$$b(y,z) = a_r^2(y,z) + a_s^2(y,z)$$

background

$$v(y,z) = a_r(y,z)a_s(y,z)\left|q(y,z)\right|\exp(i\varphi_{total})$$

visibility

The functions ***b(y,z)*** and ***v(y,z)*** can be reconstructed from complex interferograms using ***FFT*** approach.

In order to be able to compensate for ***typical errors*** both in the ***phase shift*** as well as the ***amplitude*** reconstruction, the ***reference interferograms*** comes very handy.

In the generalizations published so far it was ***silently assumed*** that two interfering parts of the ***diagnostic*** beam would be ***exactly*** the ***same***. More precisely, having exactly the same ***(y,z) structure*** in the ***interference plane***.

This could be, in principle, achieved (after a ***very careful setup***) for interferometers with an ***amplitude*** division (e.g., ***Michelson***, ***Mach-Zehnder***). In the case of the ***phase front*** division (e.g., ***Nomarski***) it is not possible at all.

Therefore, a ***new approach*** needs to be ***invented*** for the purpose of the ***most precise reconstructions*** even in the case of a ***not very high quality*** of the ***diagnostic beam***.
It can be done. Provided the ***stability*** of the diagnostic beam between the ***reference*** and the ***signal*** shots is ***sufficiently high***. As well as the ***interferometer setup stability***.

AMPLITUDE EFFECT ANALYSIS

Let us denote the ***signal*** and the ***reference*** part of the ***diagnostic*** beam in the case of the ***reference shot*** (no signal) by the lower index ***0***. In that case the ***effect of the object*** on the ***amplitude*** of the ***signal part*** of the diagnostic beam – ***f(y,z)*** - can be expressed the following way:

$$a_s(y,z) = f(y,z)\, a_{s0}(y,z)$$

Denoting as ***s(y,z)*** the ***ratio*** between the ***reference*** and the ***signal*** part of the ***diagnostic*** beam recorded intensities (the ***reference*** shot)

$$s(y,z) = \frac{I_{r0}(y,z)}{I_{s0}(y,z)} = \frac{a_{r0}^2(y,z)}{a_{s0}^2(y,z)}$$

the following ***general solution*** can be found:

$$f(y,z)=\sqrt{\frac{1}{p}\frac{b(y,z)}{b_0(y,z)}\left[1+s(y,z)\right]-s(y,z)}$$

This is the ***most general solution*** which will turn into the already published ***less general solution*** (case ***s(y,z) = 1***):

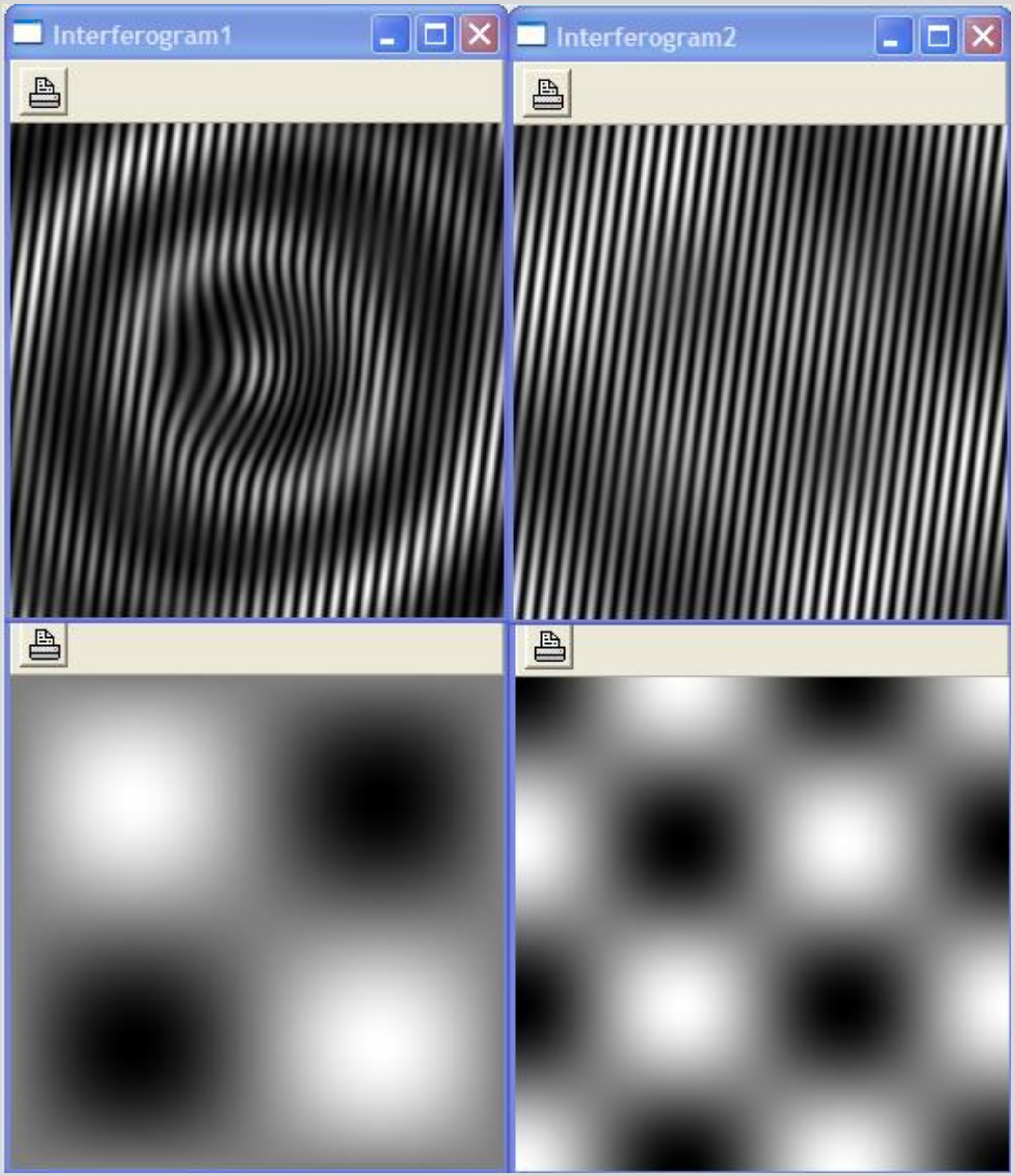
$$f(y,z)=\sqrt{\frac{2}{p}\frac{b(y,z)}{b_0(y,z)}-1}$$

Here also the parameter ***p*** was introduced as the ***ratio*** between the corresponding ***energy values*** of the ***signal*** and the ***reference*** shots (energies will vary in practice).

$$\varphi_p(y,z) = \arctan \frac{\operatorname{Im} \dfrac{v(y,z)}{v_0(y,z)}}{\operatorname{Re} \dfrac{v(y,z)}{v_0(y,z)}} - \varphi_{serr}(y,z)$$

$$\left| q(y,z) \right| = \frac{\dfrac{1}{p} \left| \dfrac{v(y,z)}{v_0(y,z)} \right|}{\sqrt{\dfrac{1}{p} \dfrac{b(y,z)}{b_0(y,z)} \left[1 + s(y,z) \right] - s(y,z)}}$$

COMPLETE SET OF ALL 4 DATA STRUCTURES



Complex Interferogram

Reference Interferogram

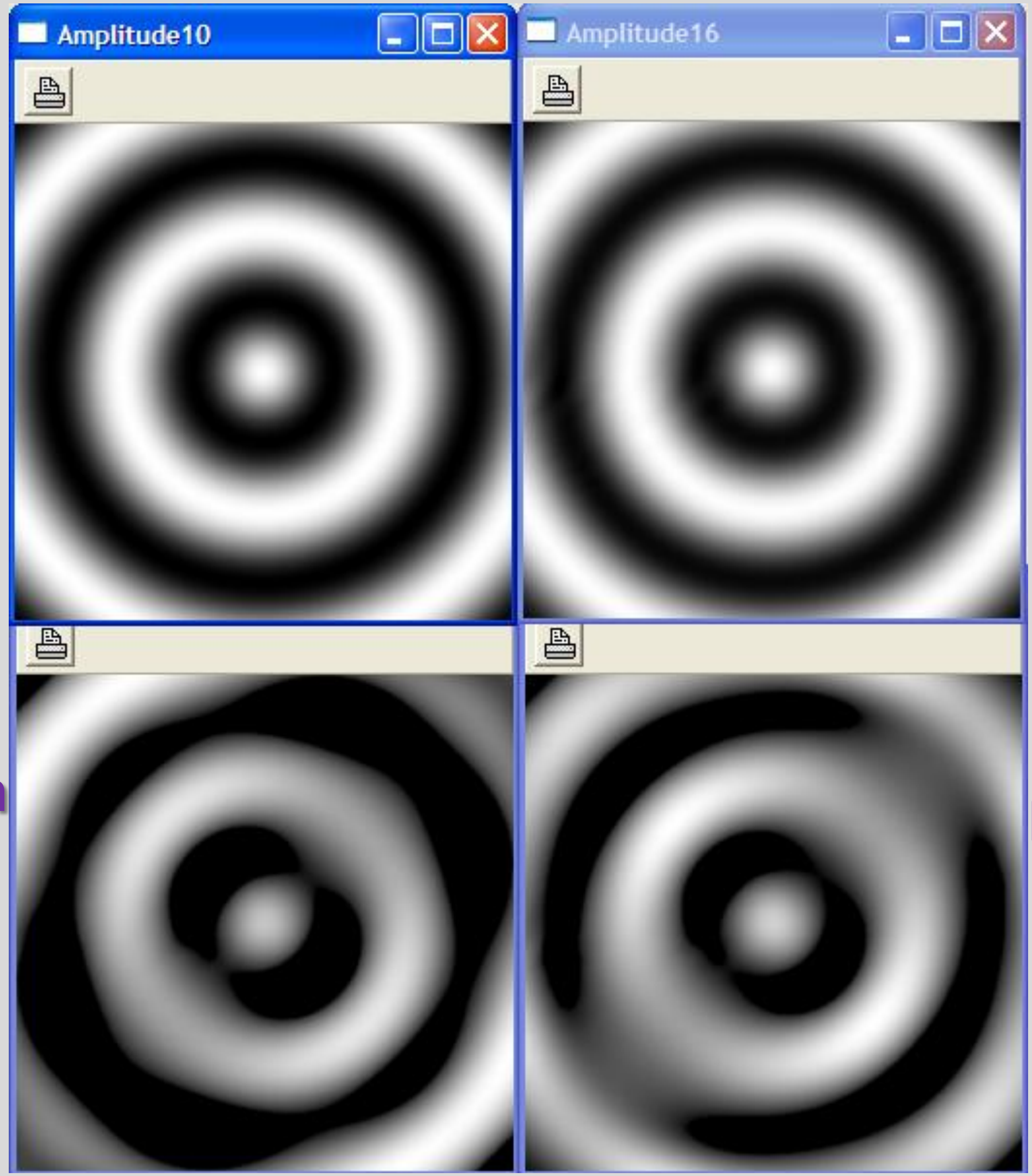
Diagnostic Beam Signal Part Intensity

Diagnostic Beam Reference Part Intensity

ANALYSIS OF THE AMPLITUDE

Original Amplitude

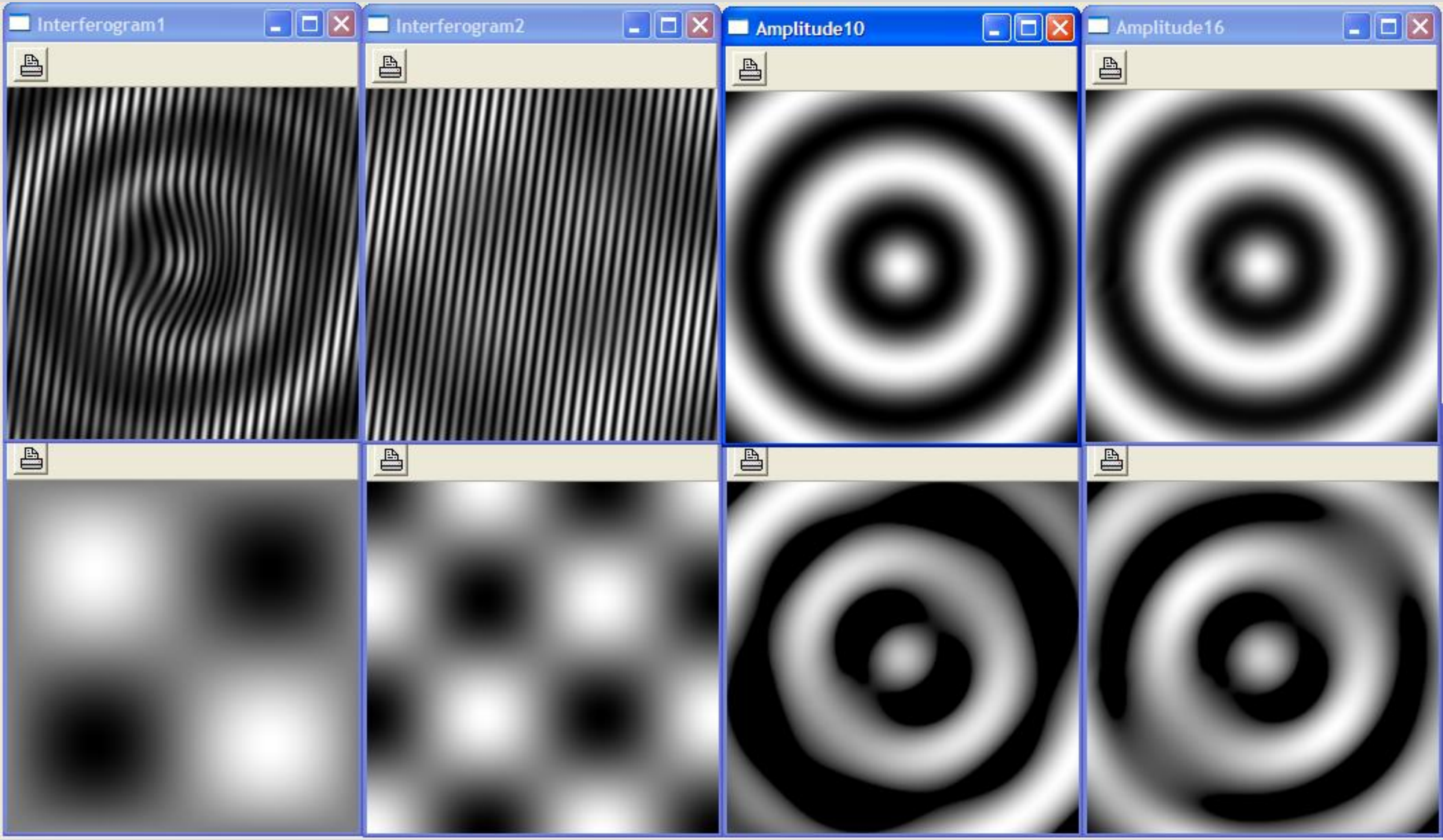
Amplitude Reconstruction using all 4 Data Structures



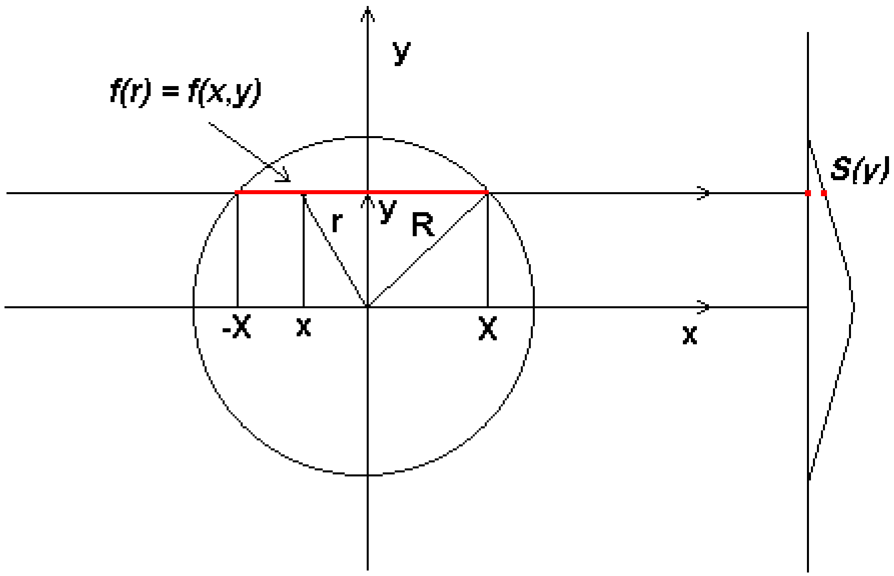
Amplitude Reconstruction from Complex and Reference Interferogram

Amplitude Reconstruction from Complex Interferogram only

AMPLITUDE RECONSTRUCTION OVERVIEW



Basic Principles of ABEL INVERSION



$$r = \sqrt{x^2 + y^2}$$

$$\frac{dr}{dx} = \frac{x}{\sqrt{x^2 + y^2}} = \frac{x}{r}$$

$$x = \sqrt{r^2 - y^2}$$

(for $x = X \Rightarrow r = R$)

$$S(y) = \int_{-X}^{X} f(x,y)dx = 2\int_{0}^{X} f(x,y)dx$$

$$dx = \frac{rdr}{x} = \frac{rdr}{\sqrt{r^2 - y^2}}$$

(for $x = 0 \Rightarrow r = y$)

$$S(y) = 2\int_{y}^{R} f(r)\frac{rdr}{\sqrt{r^2 - y^2}}$$

Abel Transform Formula

$$S(y) = 2\int_{y}^{R} f(r)\frac{rdr}{\sqrt{r^2 - y^2}}$$

$$S(y) = \mathrm{A}[f(r)]$$

formal denotation

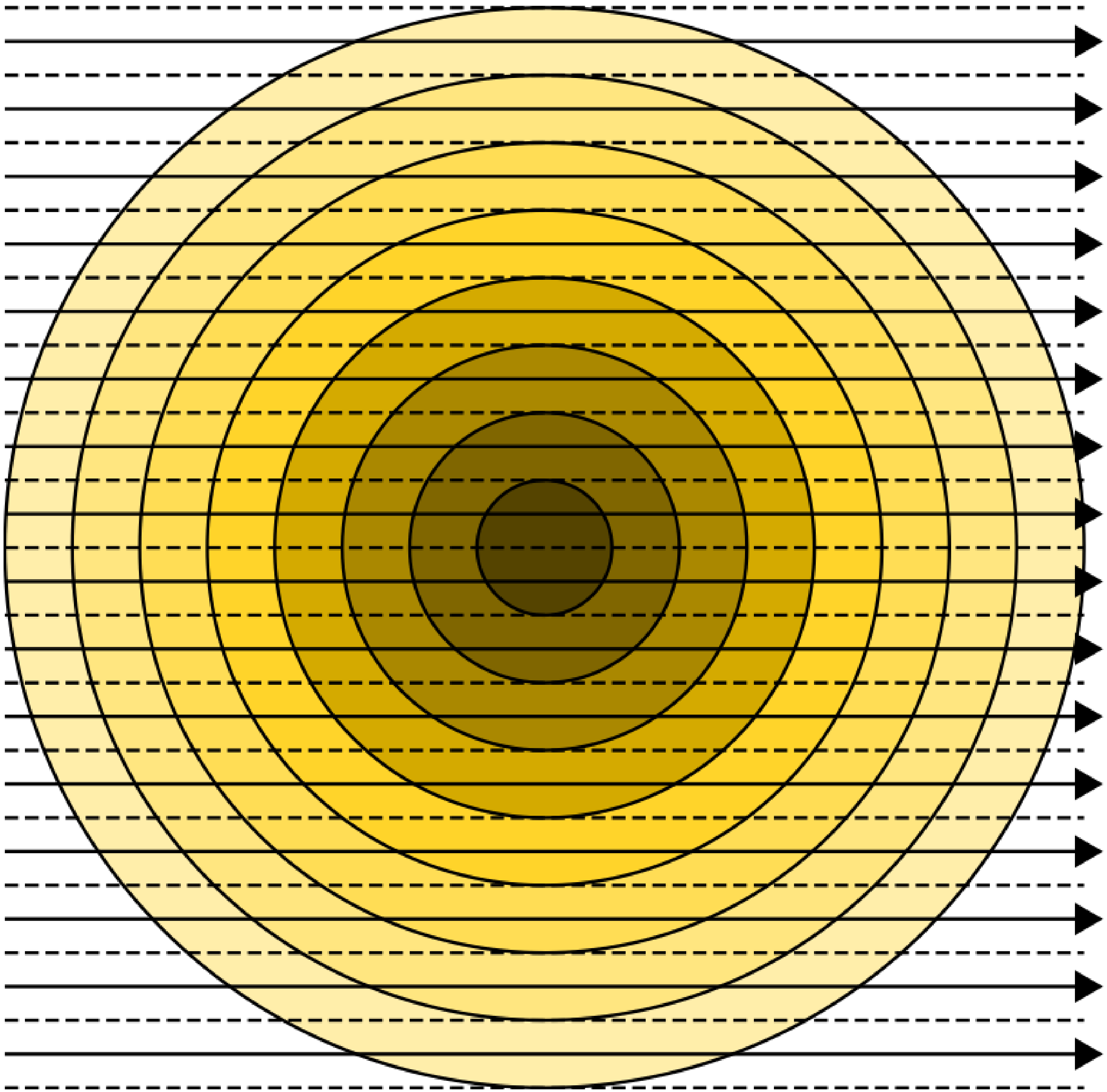
Abel Inversion Formula

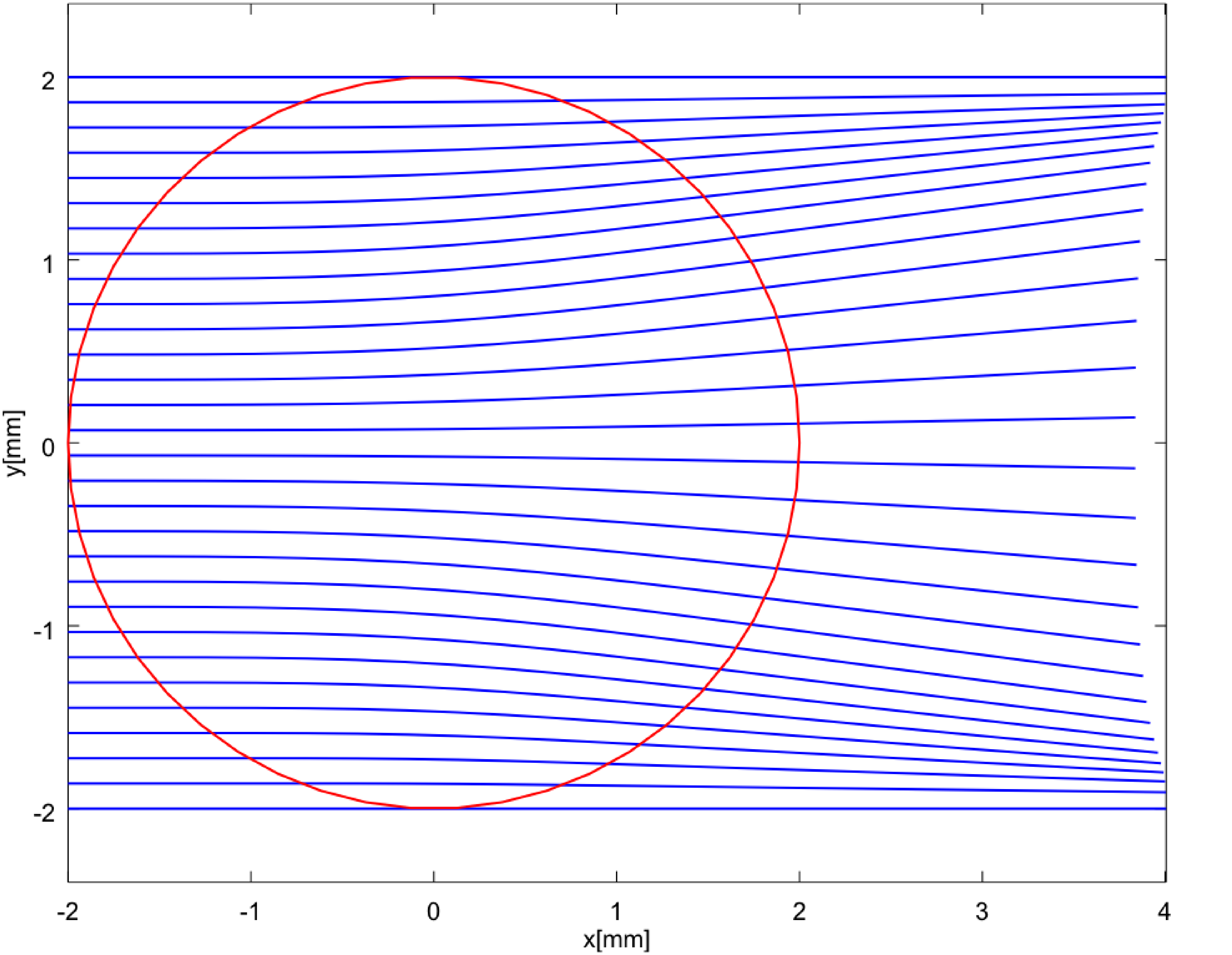
$$f(r) = -\frac{1}{\pi}\int_r^R \frac{dS(y)}{dy}\frac{dy}{\sqrt{y^2 - r^2}}$$

$$+\frac{S(R)}{\pi}(R^2 - r^2)^{-1/2}$$ **(when *S(R)* > 0)**

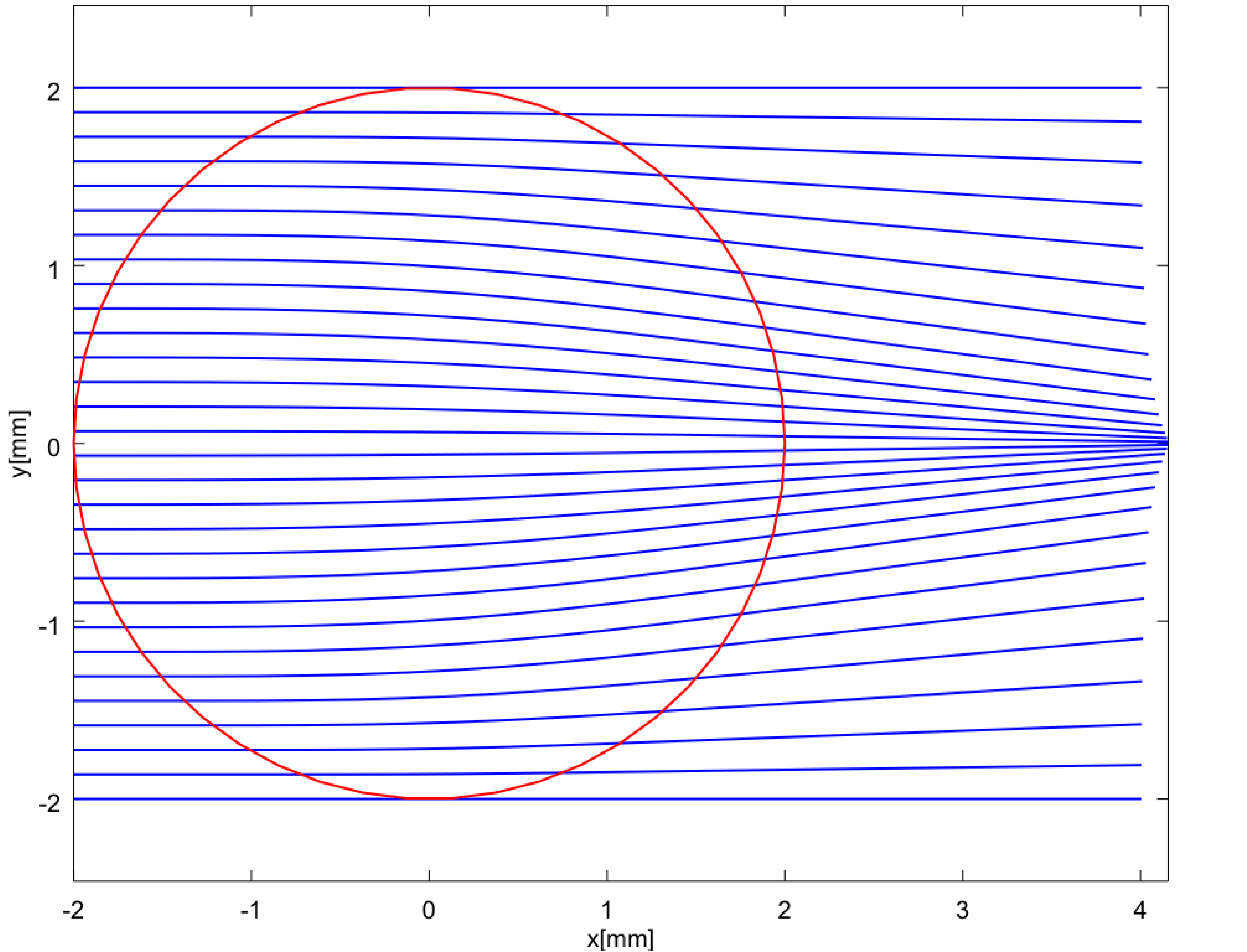
$$f(r) = \mathrm{A}^{-1}[S(y)]$$ **formal denotation**

EFFECT OF REFRACTION





2
1
0
-1
-2
y[mm]
-2
-1
0
1
2
3
4
x[mm]



2
1
0
-1
-2
y[mm]
-2
-1
0
1
2
3
4
x[mm]

Issues Connected to ABEL INVERSION

Using the ***Abel inversion*** a ***zero approximation*** to the ***index of refraction spatial profile*** is ***reconstructed*** from the ***original phase shift***.

This ***zero approximation*** contains a ***systematic error*** due to ***omitted effects*** of ***refraction*** and needs to be ***corrected***.

Both the ***phase shift*** and the ***amplitude*** are ***reconstructed*** from the ***complex interferogram***.

For this purpose the ***procedures*** and ***formulae*** presented ***earlier*** should be used:
employing the ***reference interferogram***
and separate ***intensities*** (if available).

In order to distinguish these ***particular reconstructed quantities*** from those which would gradually emerge during the ***iterative process*** they shall be addressed as

- the ***original phase shift***
- the ***original amplitude***

!!!!! Return entry point for the iteration process

Using the ***ray-tracing*** on the ***current version*** of the ***index of refraction spatial profile*** a ***new phase shift*** and ***amplitude*** are calculated.

These ***newly*** calculated ***phase shift*** and ***amplitude*** are ***compared*** with the ***original*** ones.

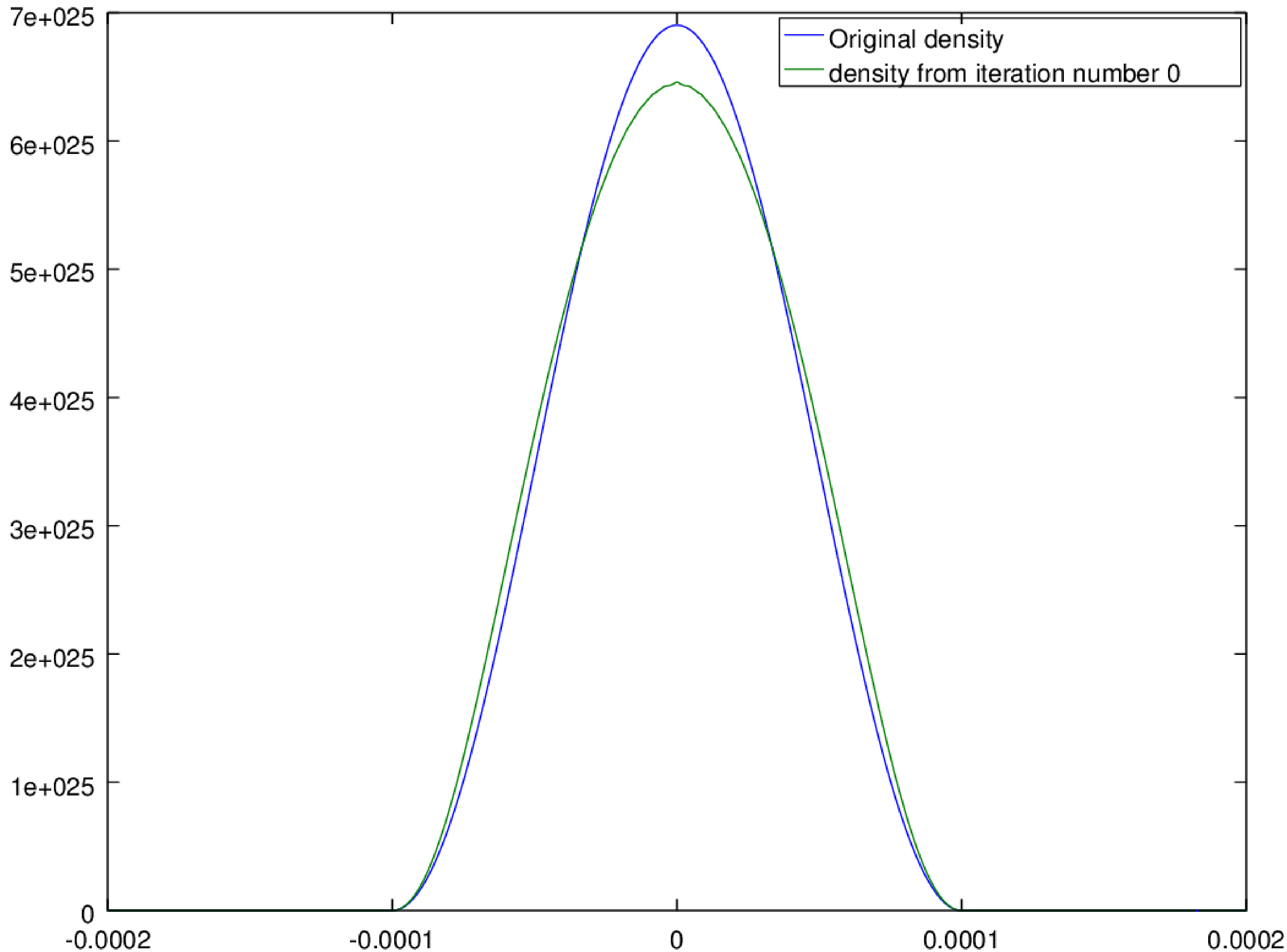
Found ***differences*** should lead to the following decisions:

– Provided these ***differences*** are ***below*** *a* given ***limit*** the ***iteration*** process is ***completed****.*

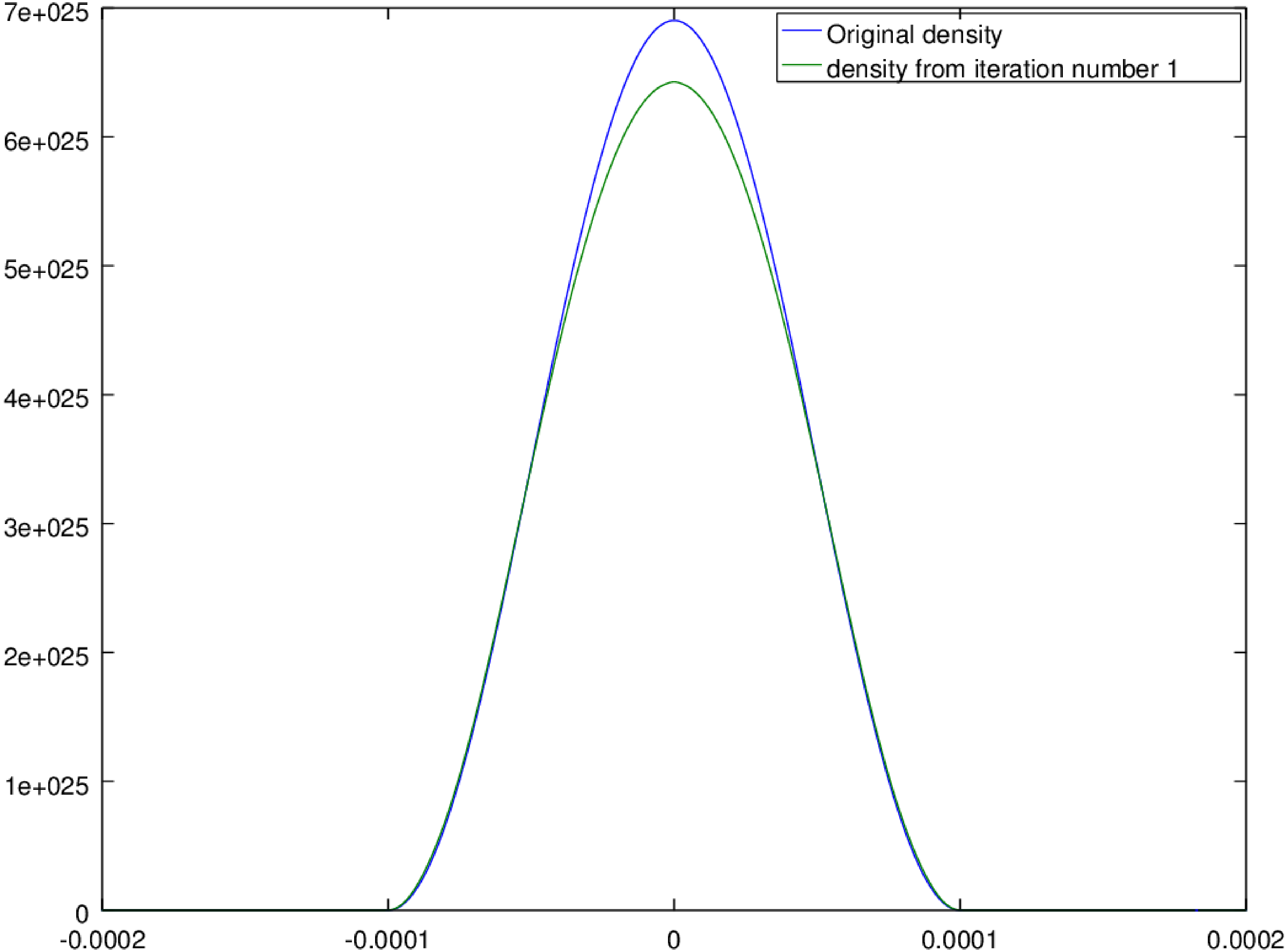
– In case these ***differences*** are ***above*** a given ***limit*** they are employed for ***corrections*** to be applied to the ***index of refraction spatial profile*** and the ***iterative*** process ***continues*** by going back to the

<u>Return entry point for the iteration process</u>

The ***algorithm*** designed for these ***corrections*** is the ***key element*** of this ***iterative*** process ***!!!***



Original density
density from iteration number 0
7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002



Original density
density from iteration number 1
7e+025
6e+025
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3e+025
2e+025
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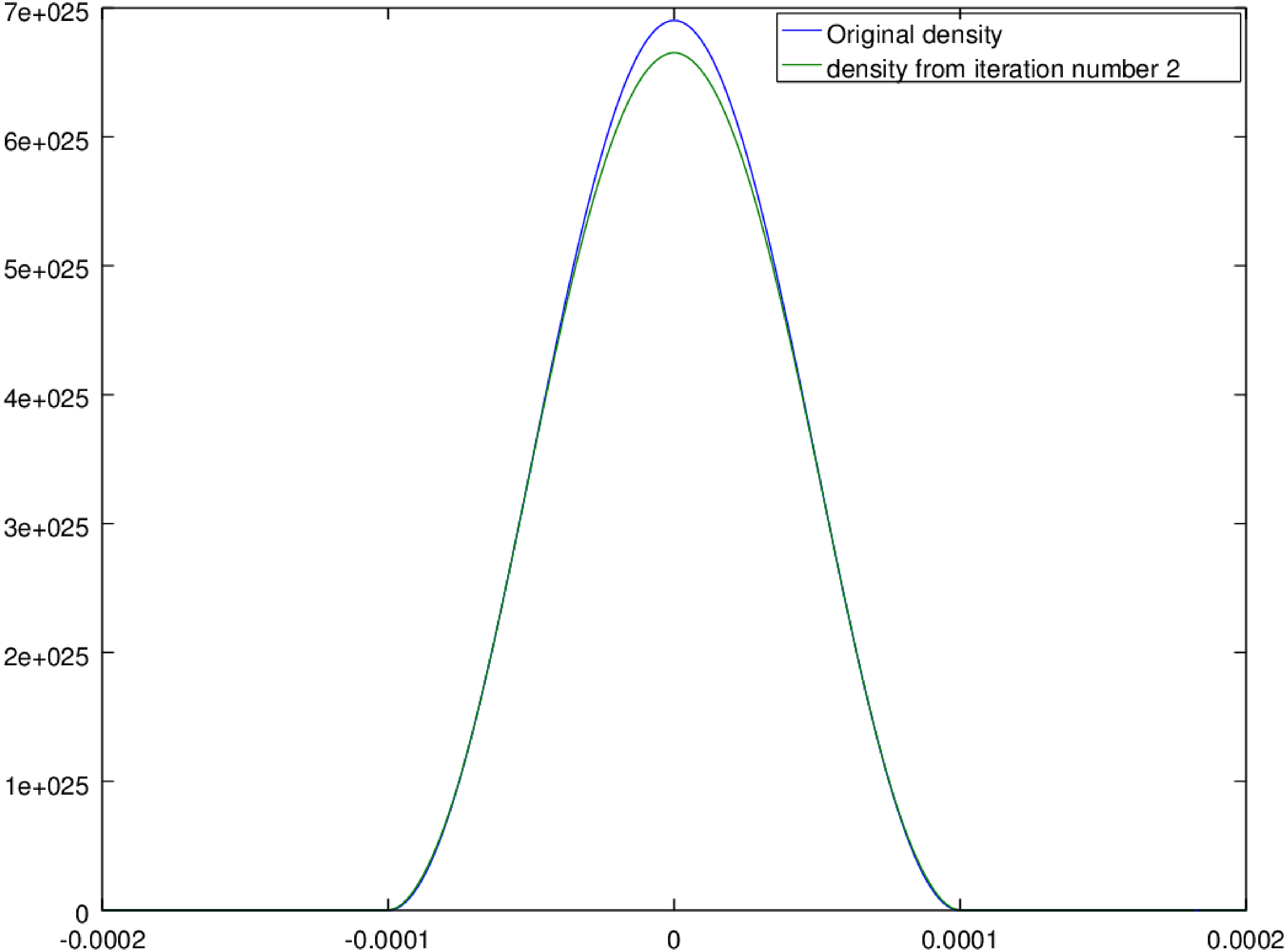
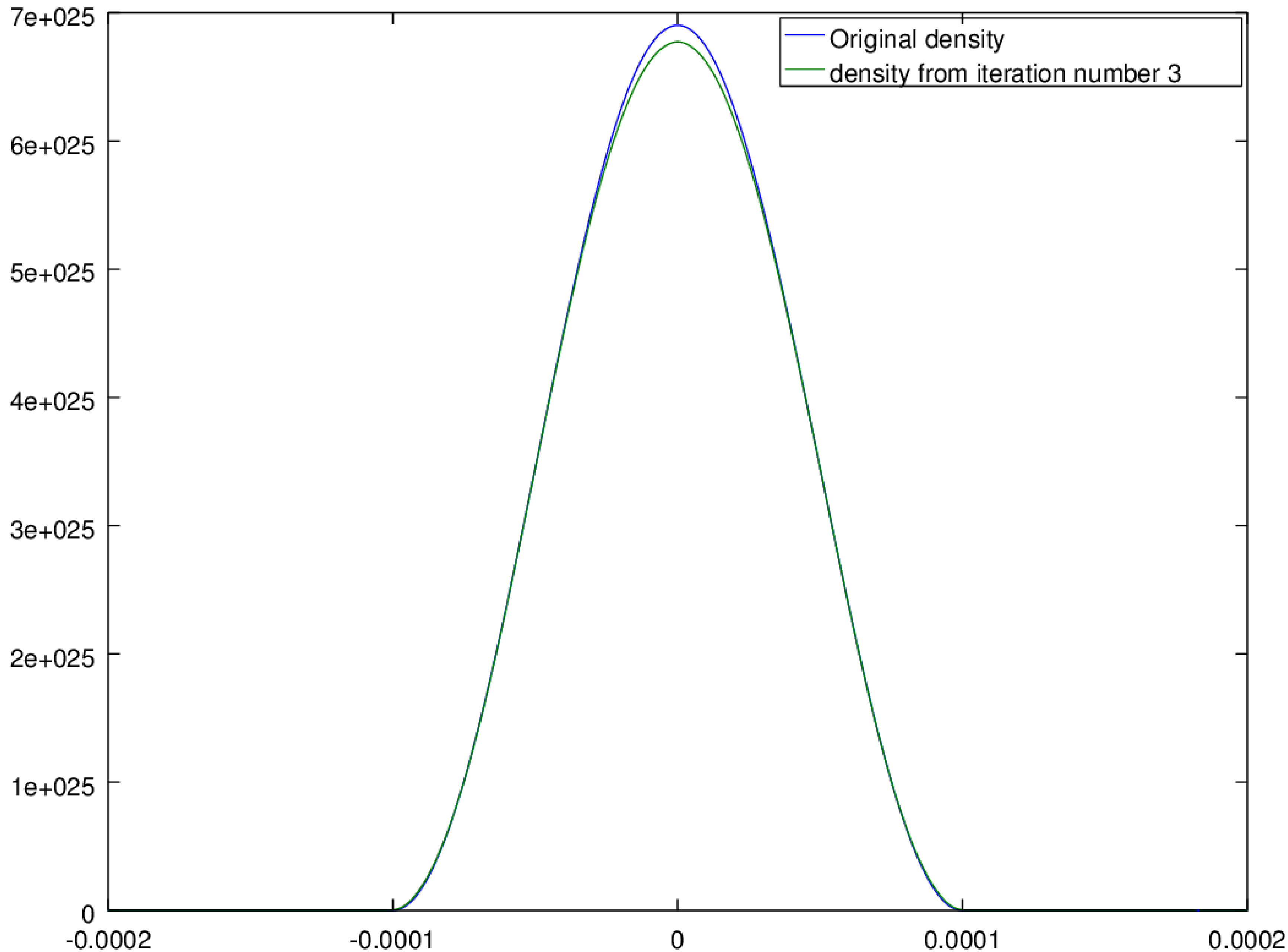
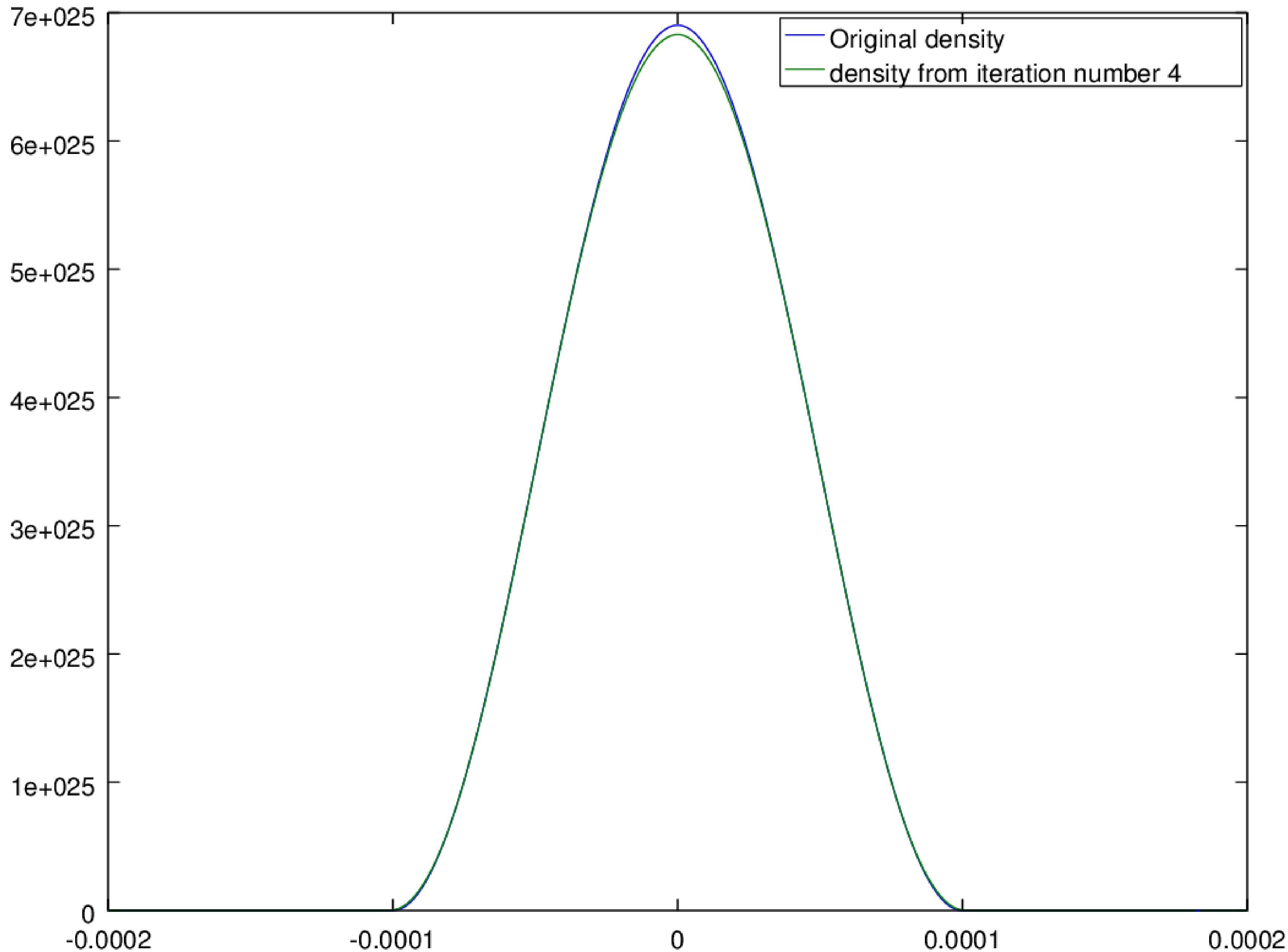
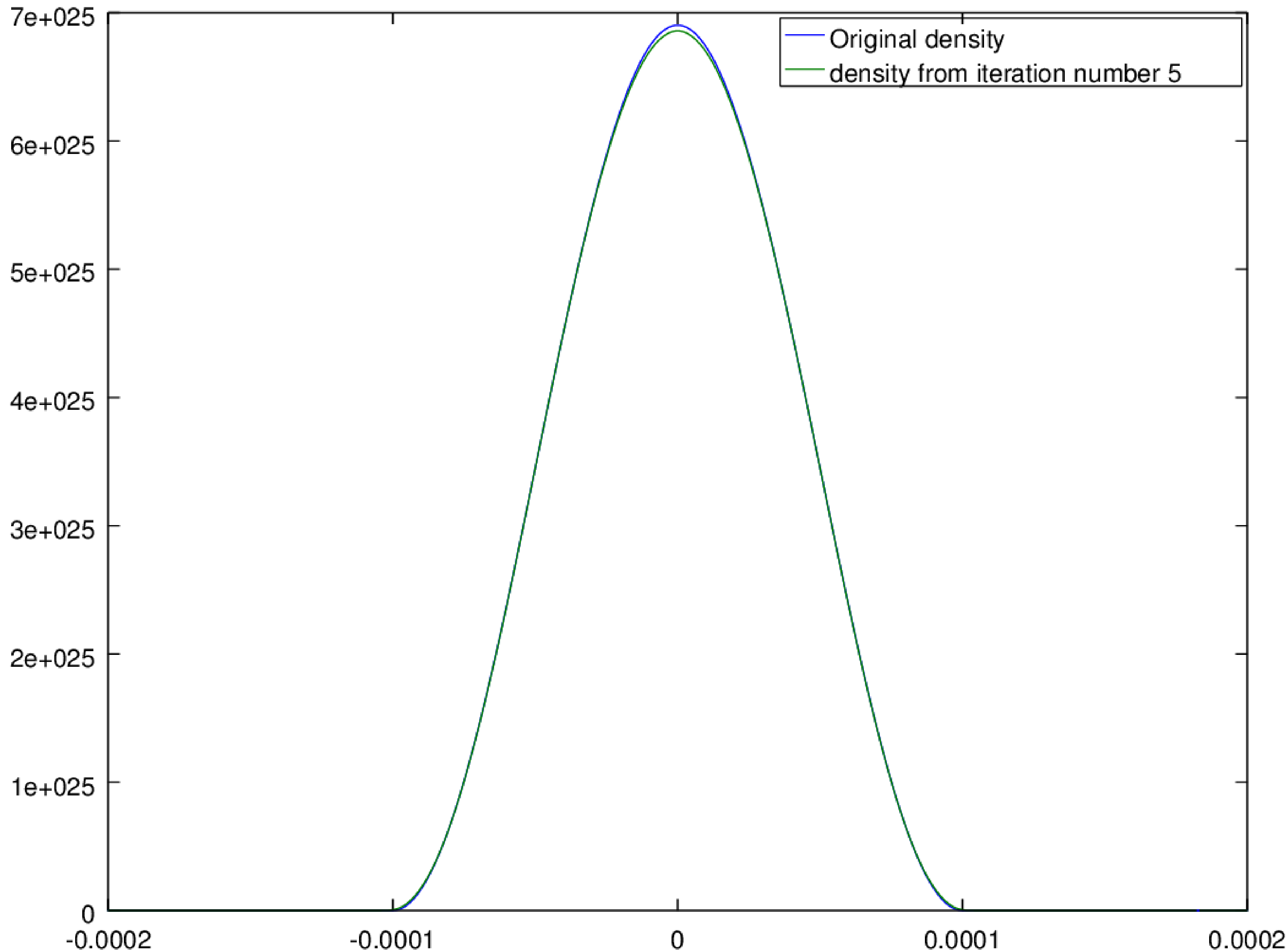
0
-0.0002
-0.0001
0
0.0001
0.0002


7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002
Original density
density from iteration number 2

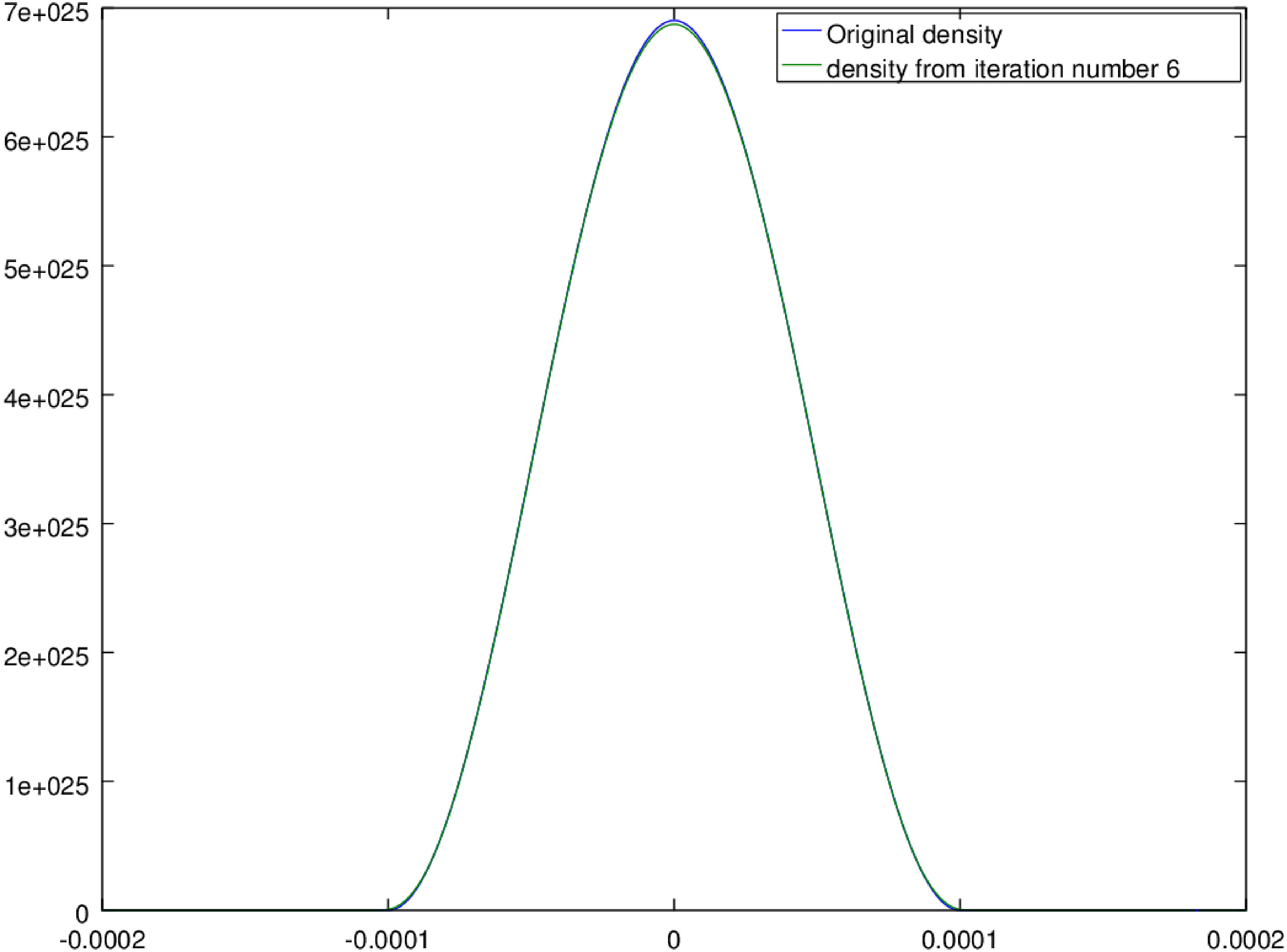


7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002
Original density
density from iteration number 3

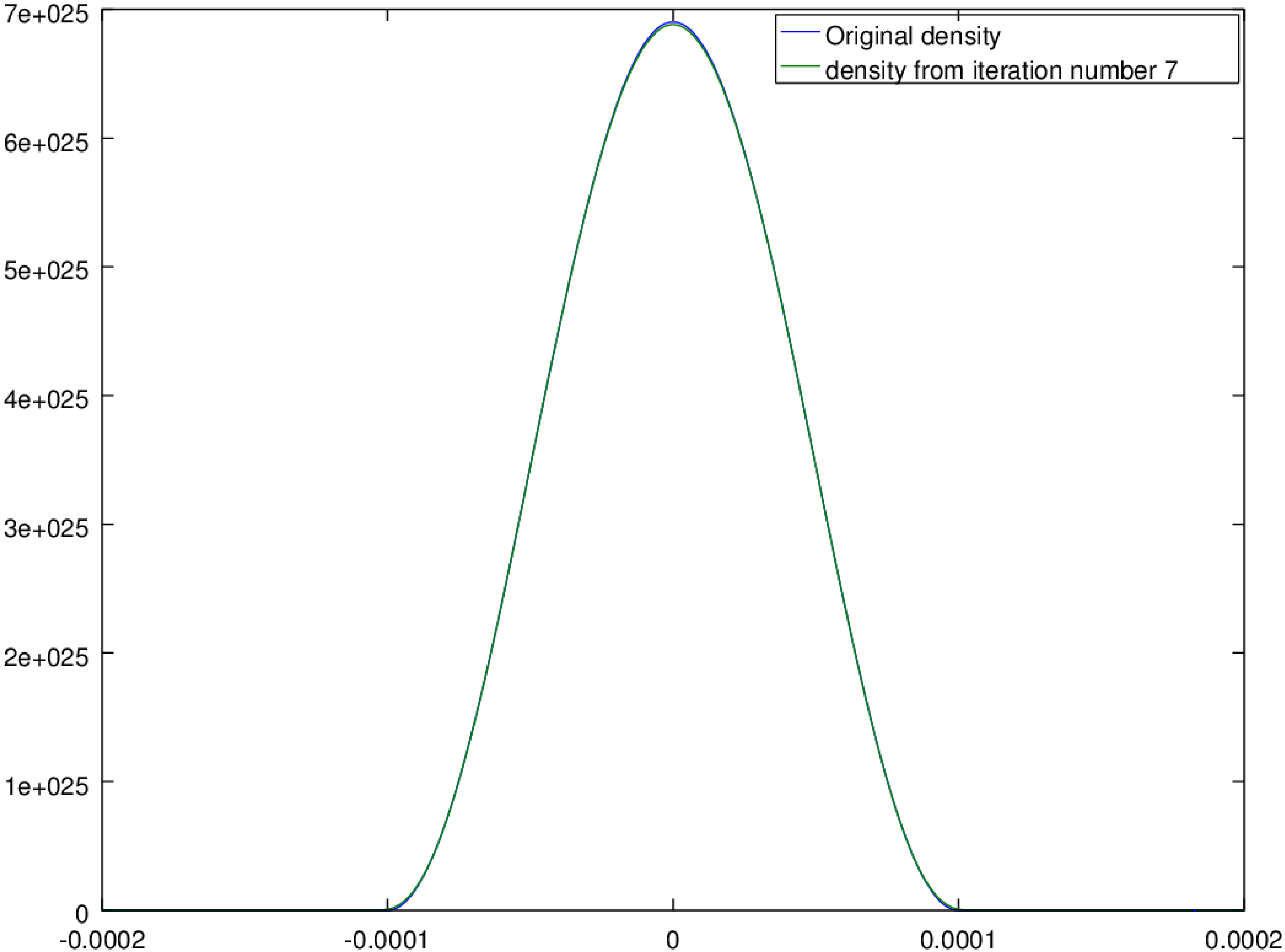

7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002
Original density
density from iteration number 4



7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002
Original density
density from iteration number 5



Original density
density from iteration number 6
7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002



Original density
density from iteration number 7
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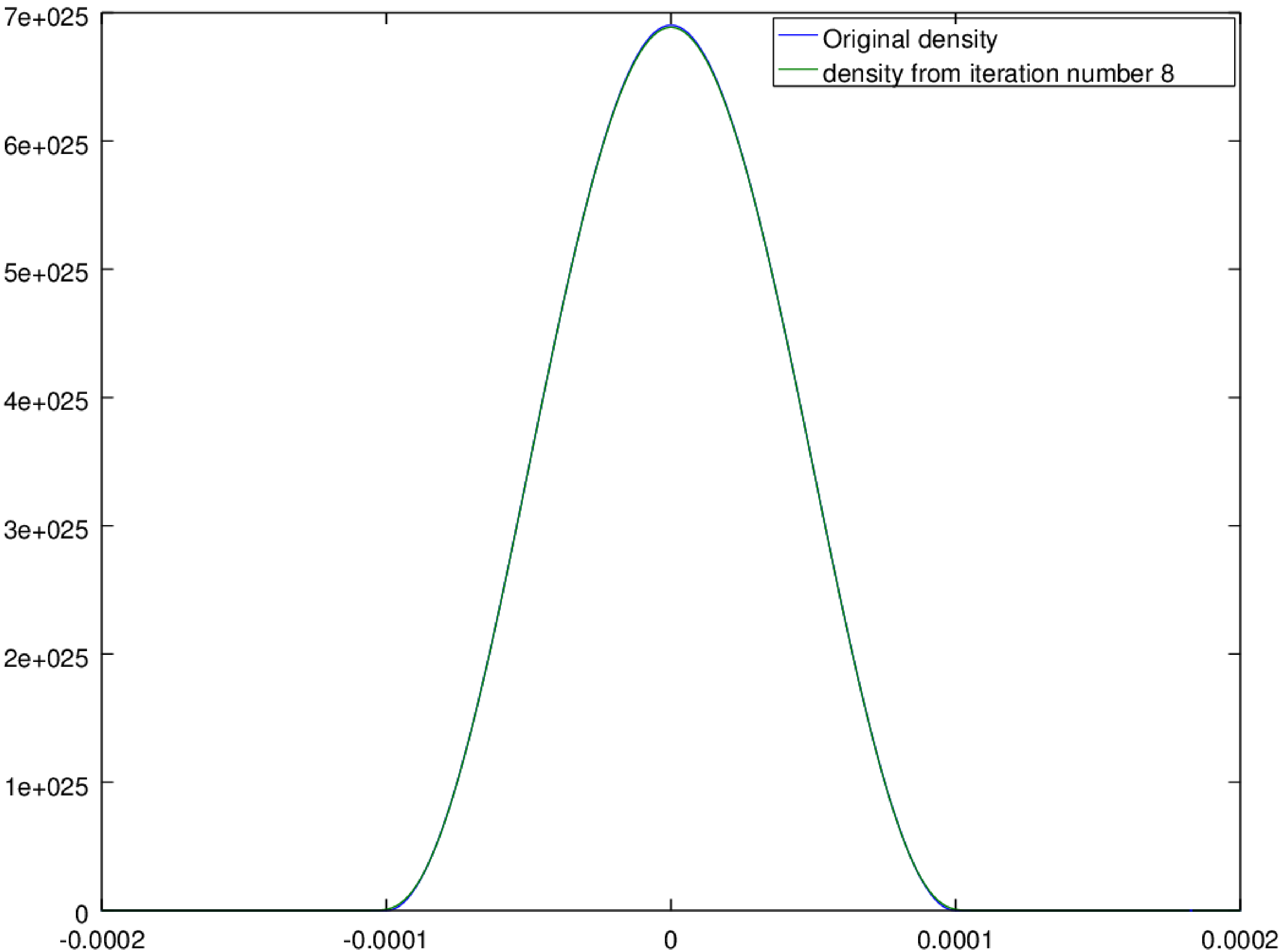
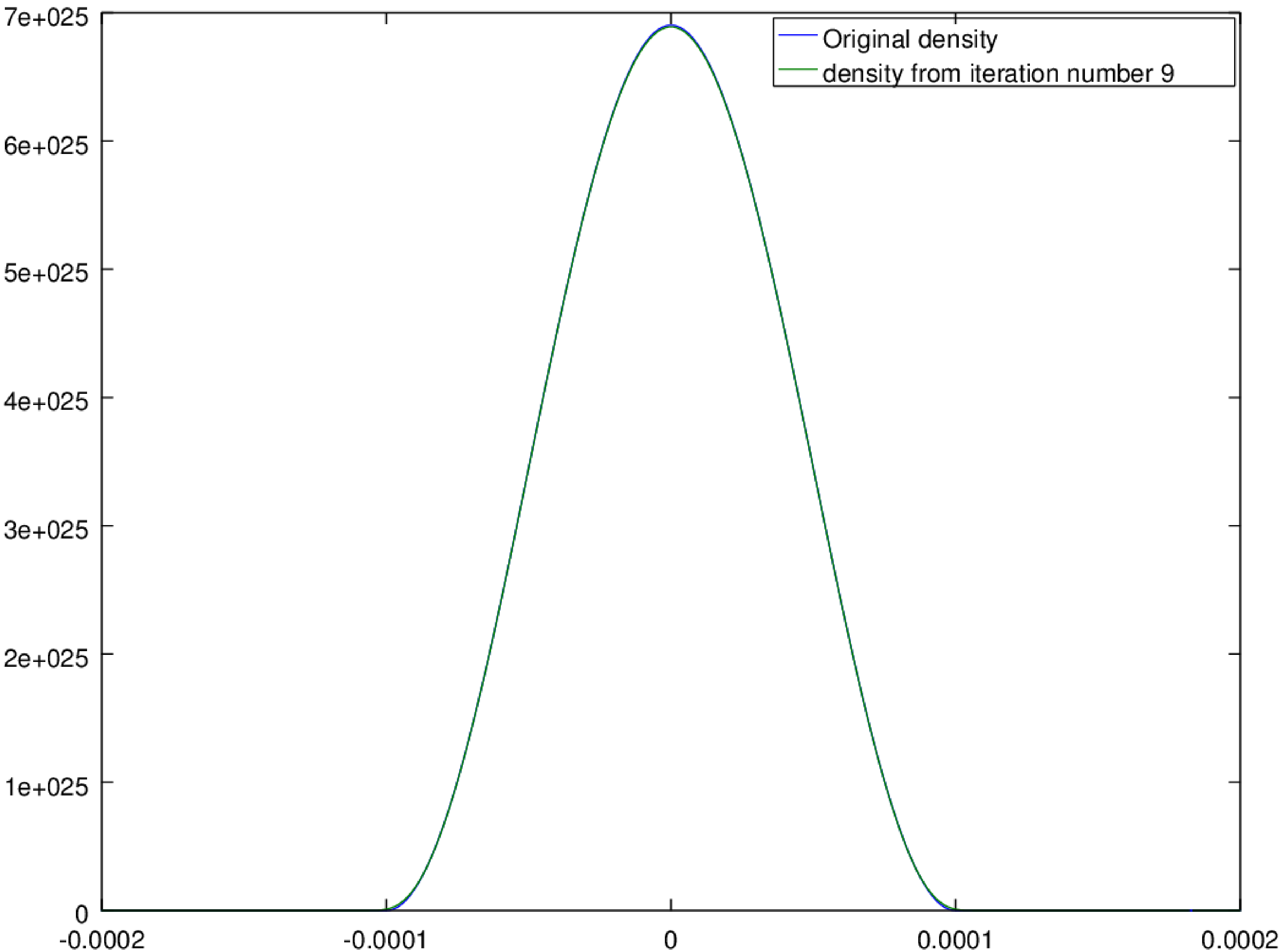
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002



7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002
Original density
density from iteration number 8



Original density
density from iteration number 9
7e+025
6e+025
5e+025
4e+025
3e+025
2e+025
1e+025
0
-0.0002
-0.0001
0
0.0001
0.0002

CONCLUSIONS

Very good ***stability*** of the ***interferometer*** as well as the ***diagnostic beam*** are ***required***. As in this case ***4 shots*** need to be taken (***signal***, ***reference*** and ***2 intensity*** structures).

If ***good enough***, the ***quality*** of the ***diagnostic beam*** as well as the ***interferometer setup*** are ***not important***.

Required information about the ***s(y,z)*** ratio can be easily obtained for interferometers with an ***amplitude*** division (***Michelson***, ***Mach-Zehnder***) where the ***signal*** and the ***reference*** part of the ***diagnostic*** beam are traveling along ***separated*** trajectories (thus easy to be ***stopped*** letting only ***one*** part to reach the detector).

In the case of the ***phase front*** division (e.g., ***Nomarski***) some care needs to be taken in order to achieve the same.

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