Introduction to Inertial Confinement Fusion (ICF)

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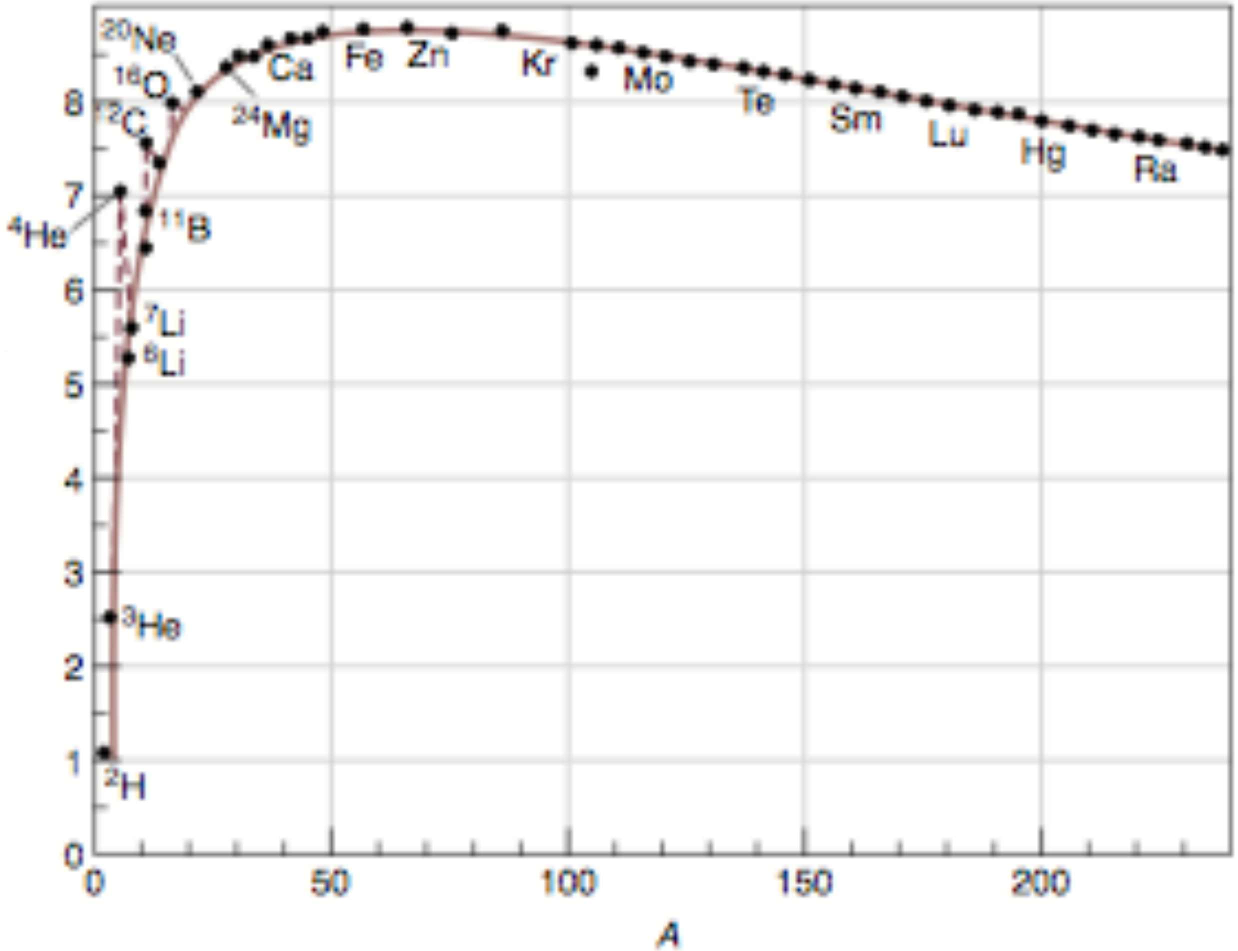
The voyage of nuclear fusion has started about 60 years ago (Sacharov, Teller, ...) and despite many progress has mainly provided disillusions...

50 years ago the laser was invented, opening the field of “Inertial Fusion” (Basov, Nuckolls, ...)

Today we are probably close to the demonstration of ignition, the scientific feasibility of fusion, which will conclude the first part of this travel.

Weizsaker semiempirical mass formula

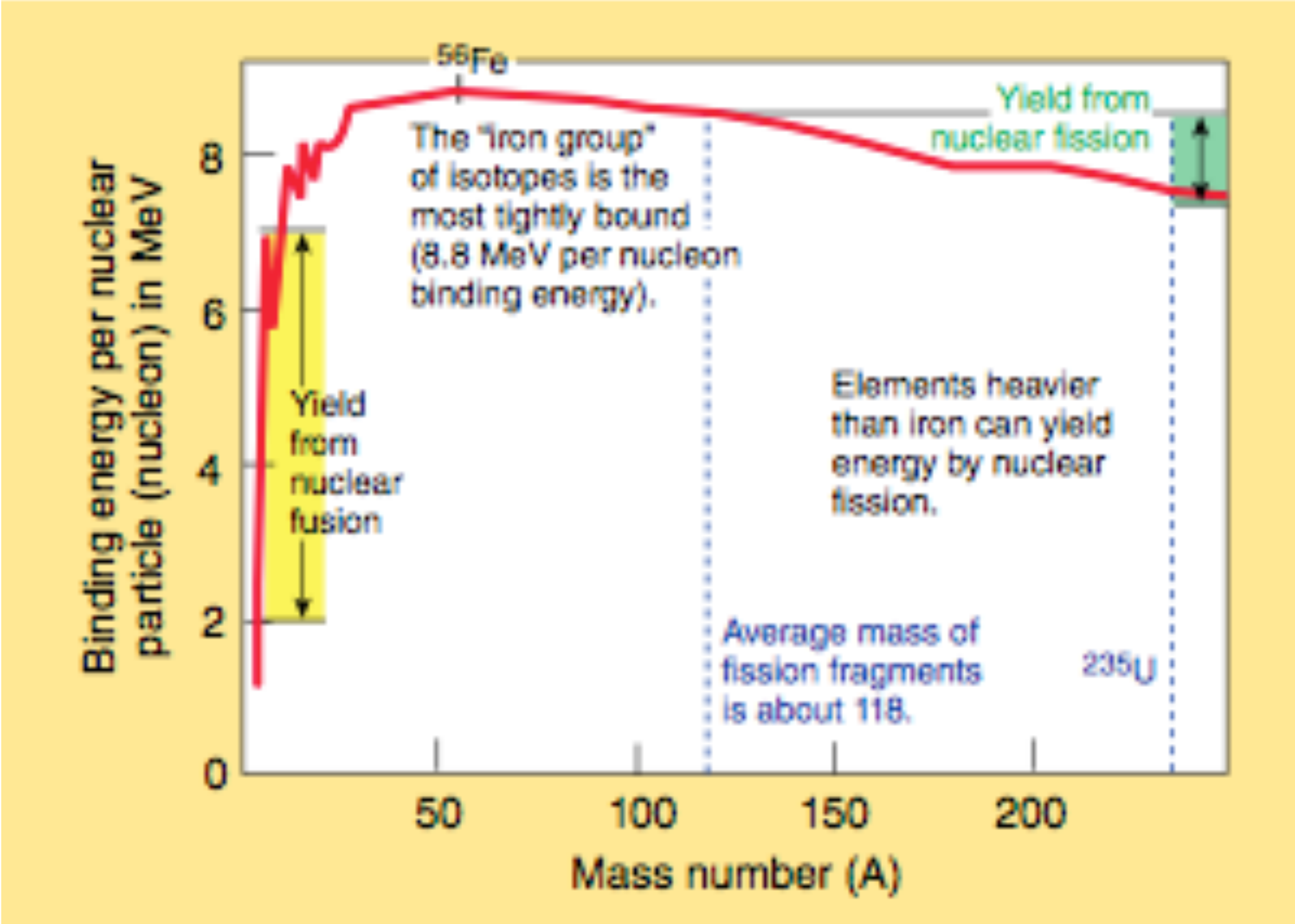


$$B_Z^A = \Delta m_Z^A c^2 / A$$

Fig. 11-10 The binding energy per nucleon versus atomic mass number *A*. The solid curve represents the Weizsäcker semiempirical binding-energy formula, Equation 11-12.

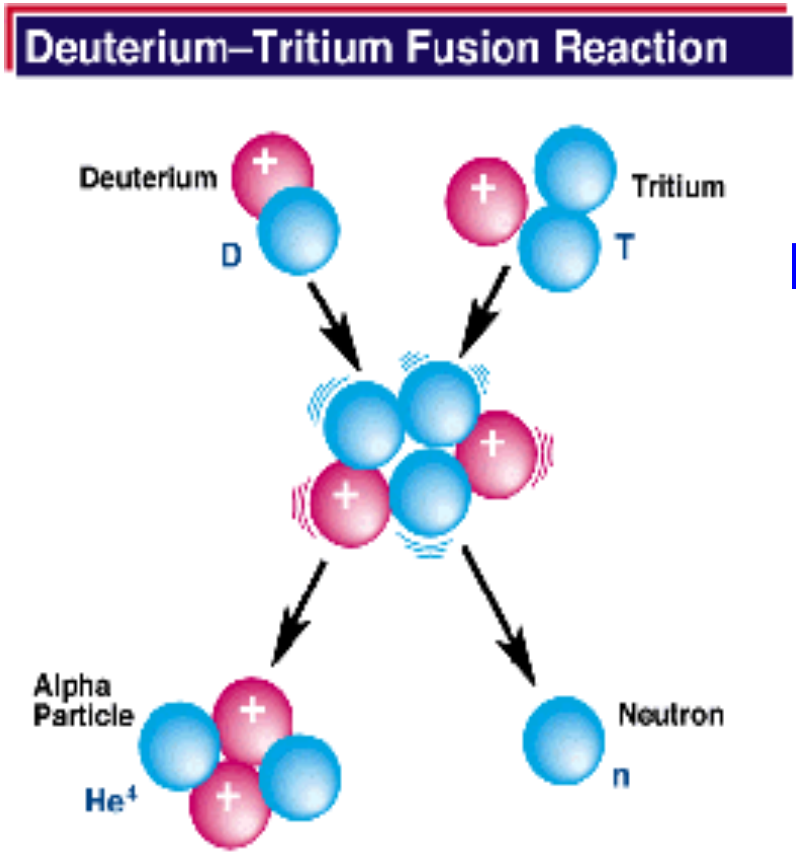
$$\Delta m_Z^A = Z m_p + (A - Z) m_n - m_Z^A .$$

Weizsaker semiempirical mass formula



$E=mc^2$

Thermonuclear Fusion:

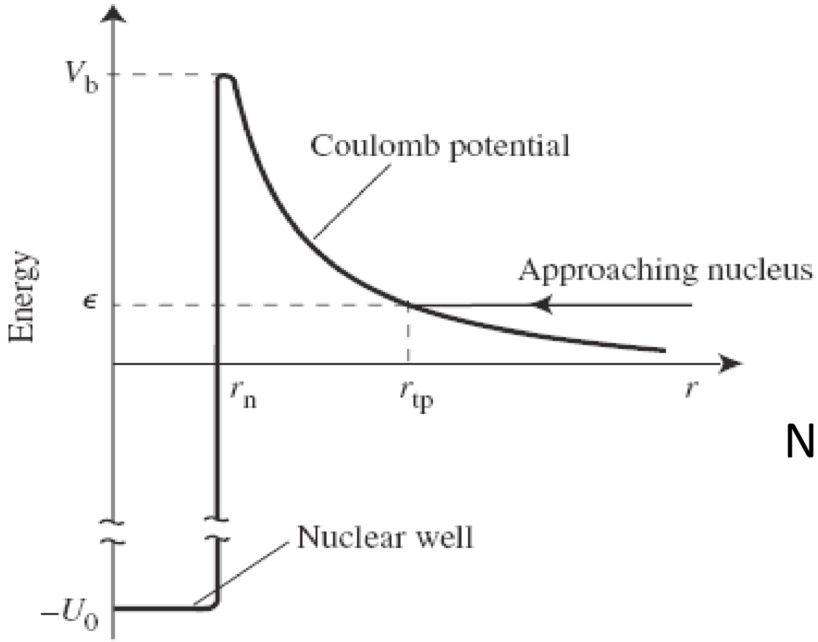


D + T → He^4 + n + 17,6 MeV

1/5 (17.6 MeV) = 3.5 MeV
for the α particle

4/5 (17.6 MeV) = 14.1 MeV
for the neutron

Coulomb repulsion:



- $$V_{Coulomb} \cong \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 (R_1 + R_2)}$$

Nuclear radius depends on *A* as

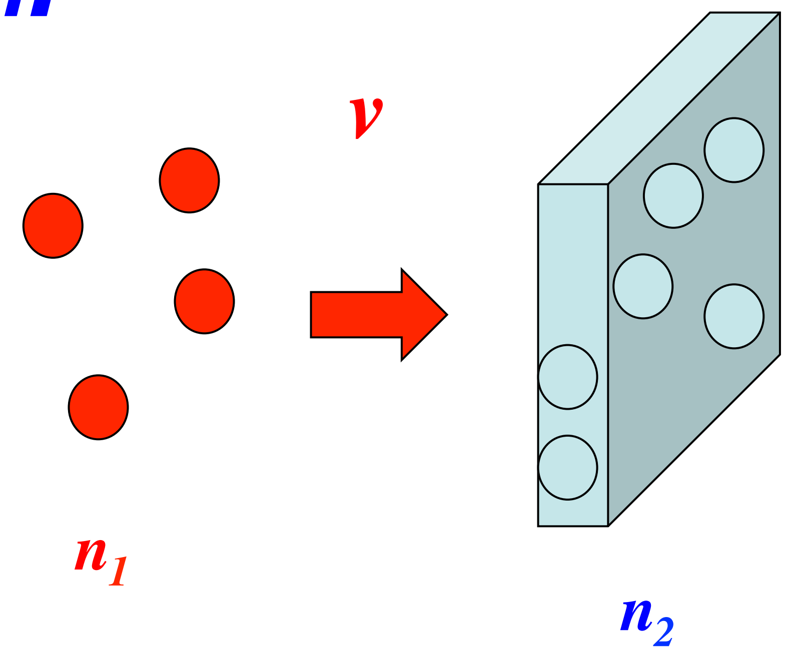
- $$R_i \cong 1.4\ 10^{-15} A_i^{1/3}$$

The energy threshold ≈ 400 KeV, i.e. T≈ 4.6 10^9 K (1 eV = 11400 K). For comparison T≈ 1.6 10^7 K at sun core.

➔ Tunnel effect

Reaction probability and cross section:

$$\frac{dn}{dt} = \sigma(v) v\, n_1 n_2$$



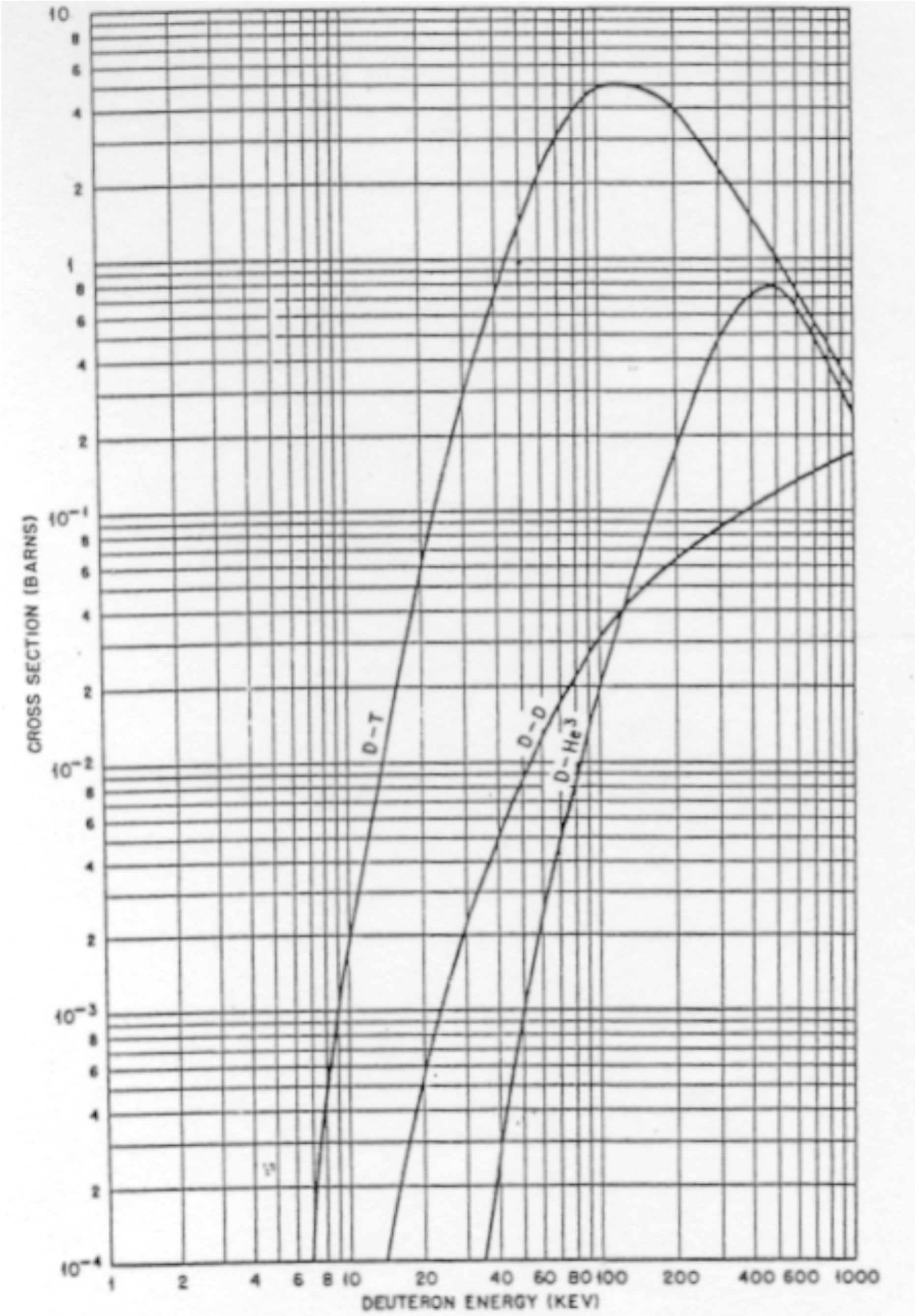
n_1, n_2 densities of particles of species 1 and 2 ($\#/cm^3$)

dn/dt number of reaction per unit volume and unit time

$n_1 v = \#/cm^2 sec$ = flux of incident particles

σ has the units of surface (cm^2)

$n_1 \sigma$ = total “covered” surface per unit volume.



Cross sections for D-T, D-D (total), and D-He³ reactions.

$D + D \rightarrow T + H^1 + 4{,}03\ MeV$

$D + D \rightarrow He^3 + n + 3{,}27\ MeV$

$D + He^3 \rightarrow He^4 + H^1 + 18{,}3\ MeV$

$D + T \rightarrow He^4 + n + 17{,}6\ MeV$

D-T fusion reaction has a larger probability and peaks at lower energy

Reaction rate:

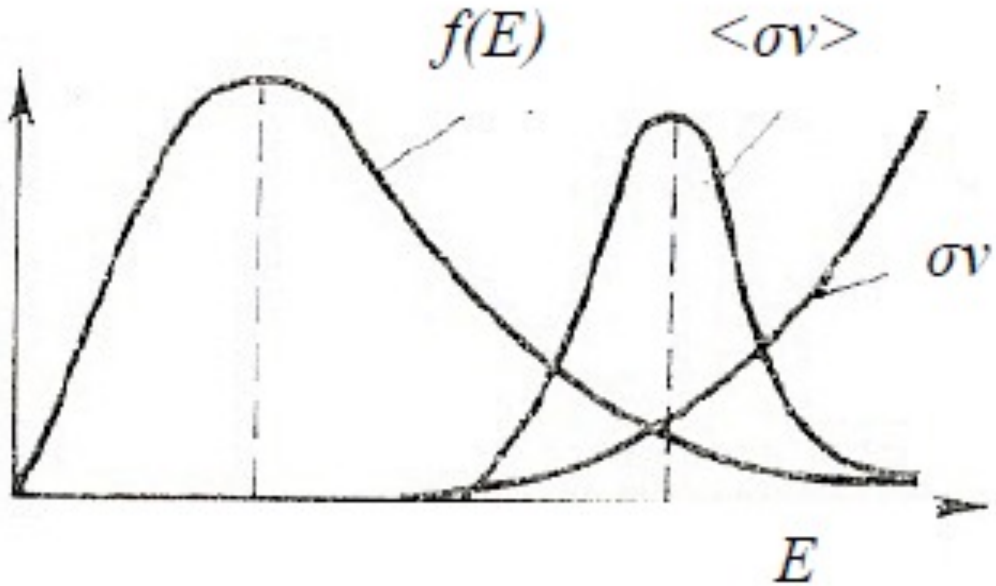
In a plasma we must average the cross section over the distribution of velocities

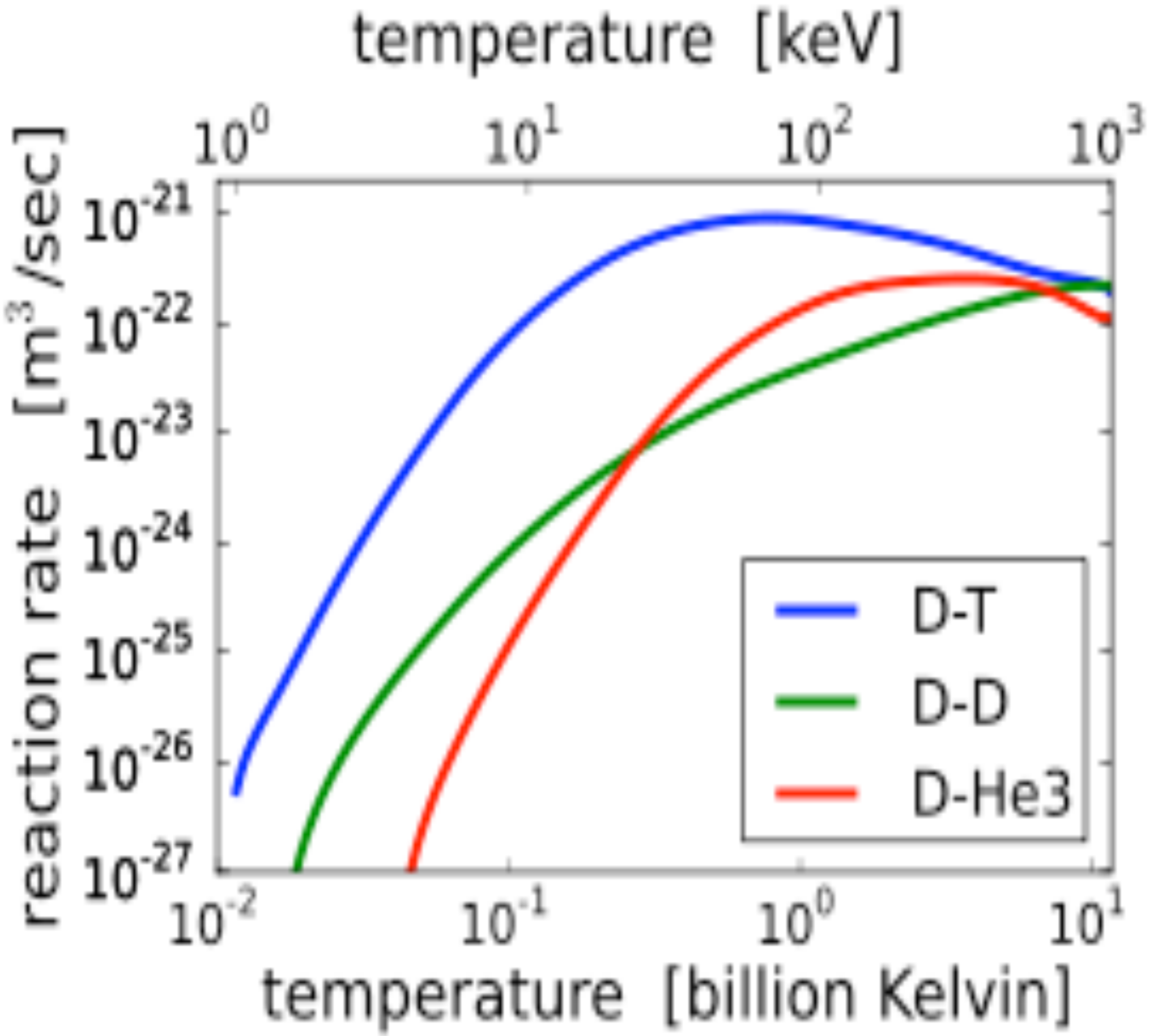
$$f_M(v) = (2\pi)^{-3/2} \exp(-m_r v^2/2T)$$

$$d\vec{v} = 4\pi v^2 dv$$

$$\langle \sigma v \rangle = 4\pi \int_0^\infty v^3 \sigma(v)\, f_M(v)\, dv.$$

At low temperature the main contribution to fusion reactions comes from ions in the distribution tail





10^9 K ≈ 10^5 eV =100 keV

Fusion needs high temperatures (T> 5 keV)

Reaction rate:

DT reaction

$$\langle \sigma_{DT} v \rangle \simeq 9.1 \times 10^{-16} \exp\left(-0.572 \left| \ln(T_{\text{keV}}/64.2) \right|^{2.13} \right) \text{ cm}^3/\text{s}.$$

3-100 keV

$$\langle \sigma_{DT} v \rangle \simeq C_{DT}^{b} T^3 = 1.1 \times 10^{-19} T_{\text{keV}}^3 \text{ cm}^3/\text{s}.$$

3-8 keV

$$\langle \sigma_{DT} v \rangle \simeq C_{DT} T^2 = 1.1 \times 10^{-18} T_{\text{keV}}^2 \text{ cm}^3/\text{s}.$$

8-20 keV

DD reaction

$$\langle \sigma_{DDn} v \rangle \simeq 2.7 \times 10^{-14} T_{\text{keV}}^{-2/3} \exp\left(-19.8 T_{\text{keV}}^{-1/3} \right) \text{ cm}^3/\text{s}.$$

3-50 keV

Plasma conditions for fusion: confinament time

Confinament time: $\tau = \dfrac{W}{P_{losses}}$

W = internal energy per unit volume (J/m^3)
P_{losses} = dissipated power per unit volume (this is mainly radiation losses due to bremsstrahlung).
τ = the time over which the system is able to keep its energy)

$$W = \frac{3}{2}\left((n_e + n_i)kT\right) = 3n_e T(eV)$$

Rate of energy production

$$P_{fusion} = n_D n_T \langle \sigma v \rangle E_\alpha = \frac{1}{4} n_e^2 \langle \sigma v \rangle E_\alpha$$

We take into account only the energy in a particles: only they remain confined in the plasma and heat it up!

LAWSON'S CRITERIUM

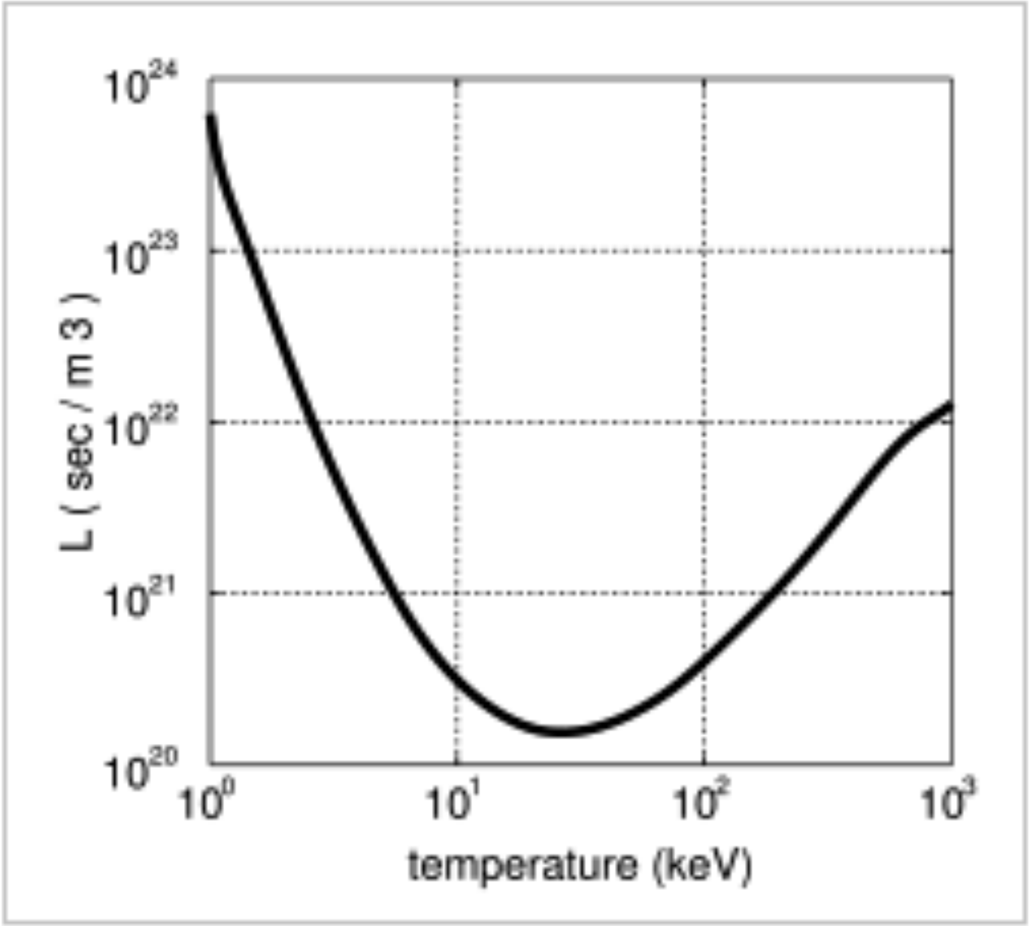
$$P_{fusion} > P_{losses} \qquad \Rightarrow \frac{1}{4} n_e^2 \langle \sigma v \rangle E_\alpha > 3 n_e T / \tau$$

$$\Rightarrow n_e \tau > 12 T / \langle \sigma v \rangle E_\alpha \equiv L$$

$\langle \sigma v \rangle$ depends on T.

[$E_\alpha = 3.5$ MeV]

$$L \equiv 12T / \langle \sigma v \rangle E_\alpha$$



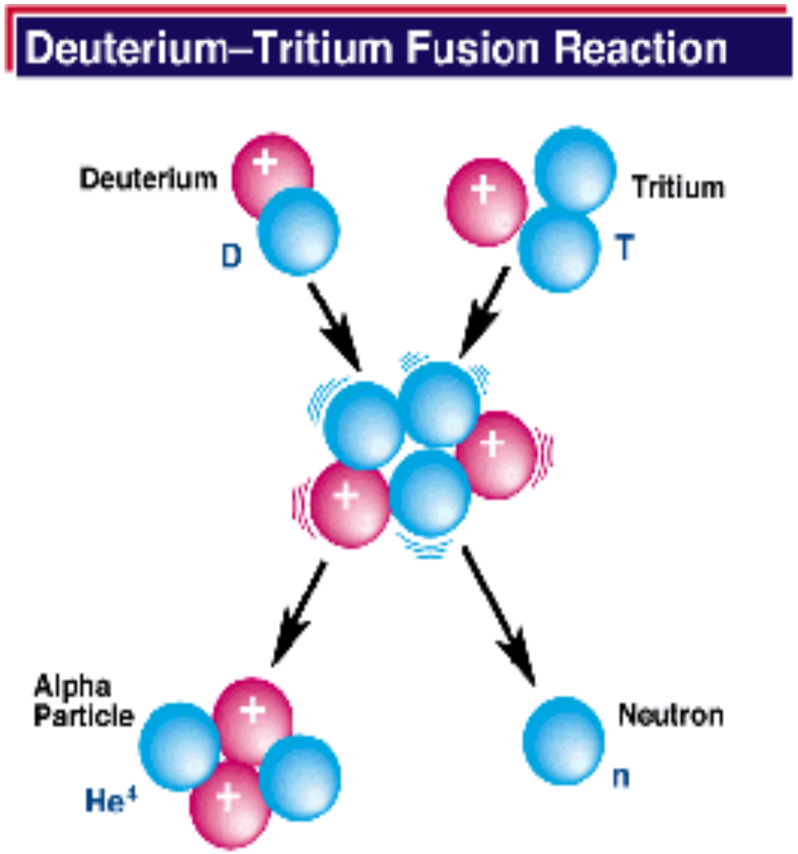
Is function of T and has a minimum at ≈ 25 keV ($2.9 10^8$ K) for $D + T \rightarrow {}^4He + n$

The deuterium-tritium L function (minimum $n_e \tau_E$ needed to satisfy the Lawson criterion) minimizes near the temperature 25 keV (300 million kelvins).

For D-T at T = 25 KeV, **Lawson's criterium** is :

$$n_e \tau > 1.5 \cdot 10^{14} \text{ s/cm}^3$$

Thermonuclear Fusion:



- **Need to have high temperatures to overcome Coloumb repulsion**

  $T_{min} \approx$ **5 - 10 keV**

- **Need to have many fusion reactions to allow for energy gain, i.e. large number of particles and/or long confinement time.**
  Lawson's criterium

  $n_e \tau \approx 1.5\ 10^{14}$ **s cm**$^{-3}$

"Triple product" $n_e \tau\ T \geq 8\ 1014$ s cm^{-3} keV

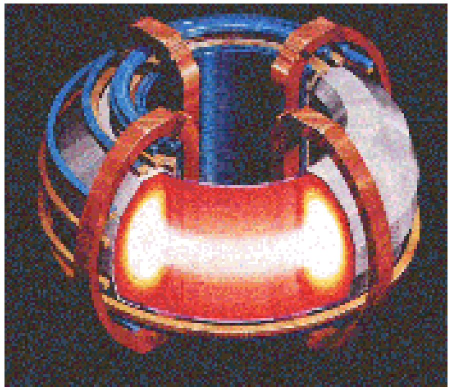
Creating conditions for fusion:

Gravitational Confinement

Heating Mechanisms:
* Compression (gravity)
* Fusion Reactions (such as the p-p chain)

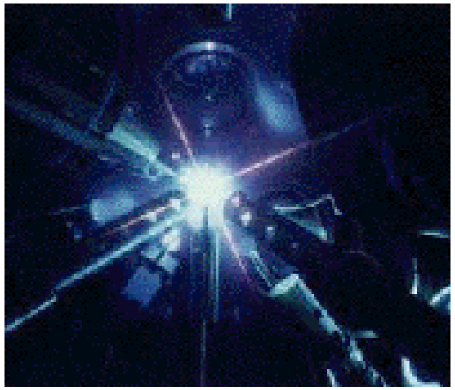
Magnetic Confinement

* Electromagnetic Waves
* Ohmic Heating (by electric currents)
* Neutral Particle Beams (atomic hydrogen)
* Fusion Reactions (D+T)



Inertial Confinement

* Compression (implosion driven by laser, or by X-rays from laser, or by ion beams)
* Fusion Reactions (primarily D+T)



Creating conditions for fusion:

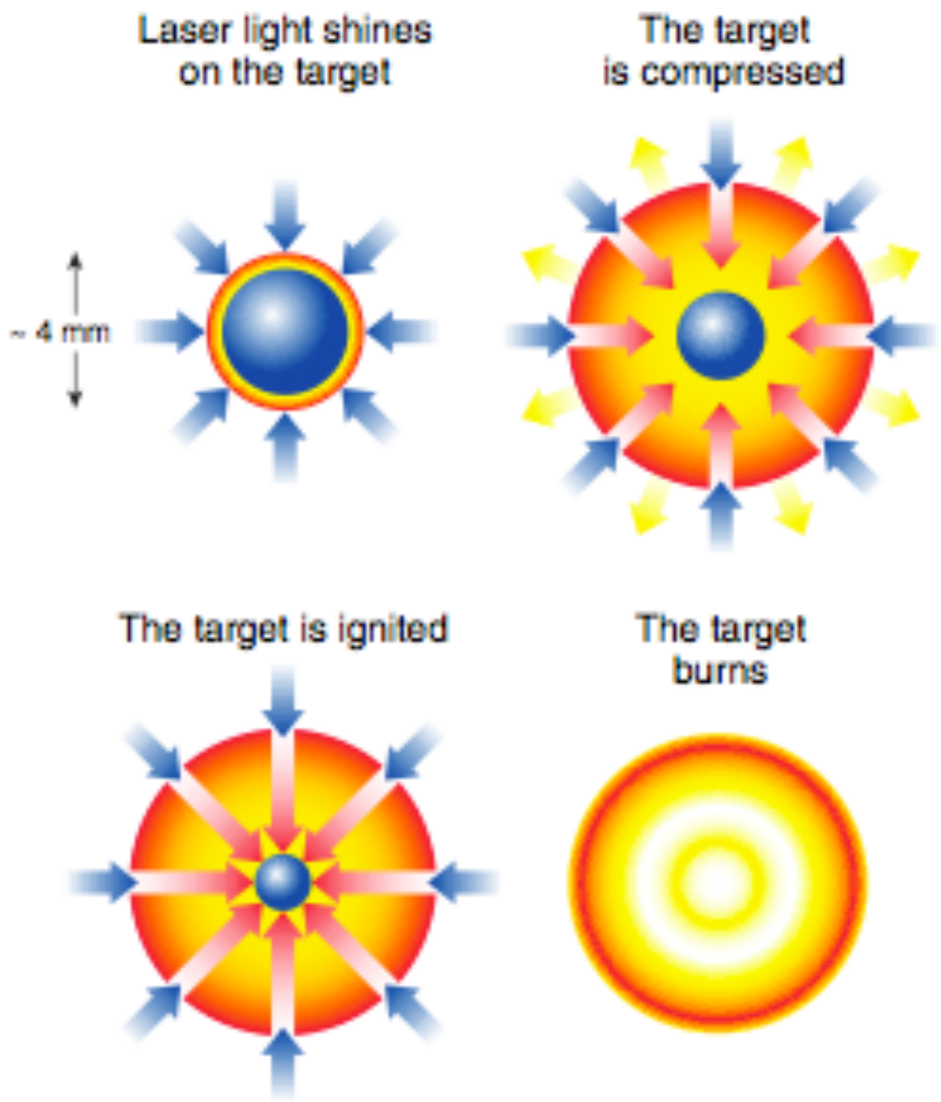
The secondary system (H-bomb) is ignited by the explosion of a “conventional” nuclear bomb

For controlled nuclear fusion:

•Need to ignite small mass of fuel

•Need to ignite with different tools!

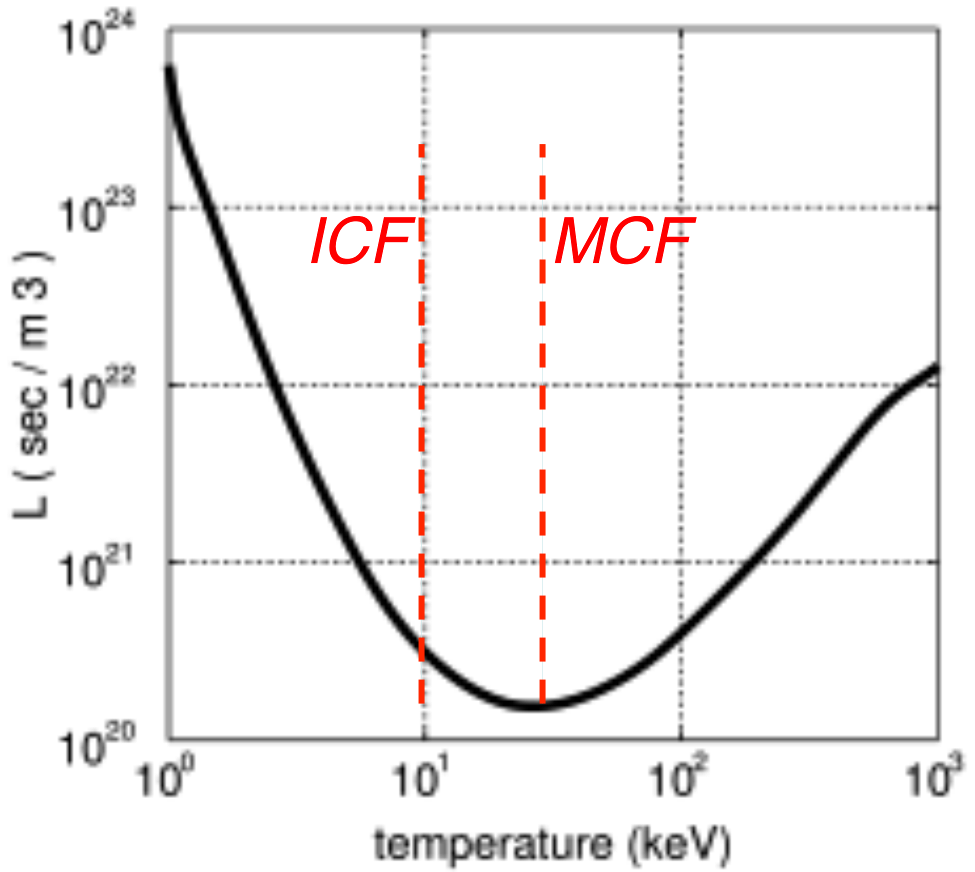
Principio del Confinamento Inerziale



Lawson's criterium for inertial fusion

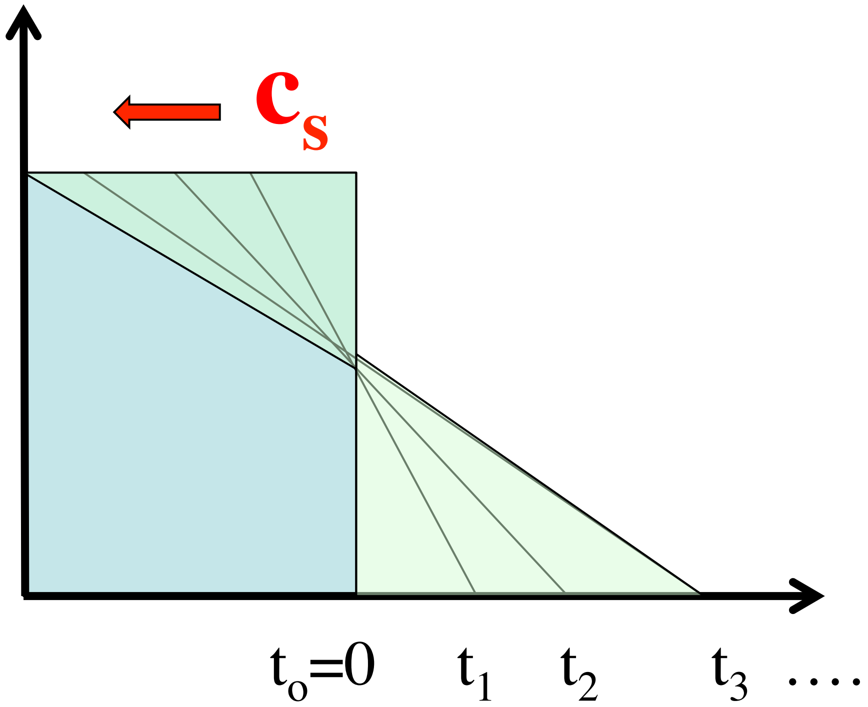
In ICFwe cannot reach 25 keV. Then

$\mathbf{n_e \tau \approx 6\ 10^{14}\ s\ cm^{-3}}$



Isothermal expansion of a gas

Rarefaction (expansion) wave
Self-similar model



$$n(x) = n_o \exp(-x/L)$$

$$L = c_s \ \mathrm{t}$$

$$c_s = (\gamma ZkT/m)^{1/2}$$

Lawson's criterium for inertial fusion

$n_e \tau \approx 6\ 10^{14}$ s cm^{-3}

Disassembly time determined by the fuels inertia

$$\tau < R / c_s \qquad \Rightarrow \quad \text{we take } \tau = R / 4c_s$$

ion sound velocity in a plasma

$$c_s = \left(\gamma Z k T_e / m_i\right)^{1/2} = 9.8 \quad 10^5 \left(\gamma Z\ T_e(eV) / \mu\right)^{1/2} cm/s$$

$$\approx 7 \quad 10^7 \quad cm/s \quad (\text{for } \ \mathrm{T} = 10\ \text{keV})$$

Notice:

in magnetic fusion the time τ expresses the *confinement of energy*

In inertial fusion it refers to the *confinement of mass*

Lawson's criterium for inertial fusion

$n_e \tau \approx 6\ 10^{14}$ s cm^{-3}

Disassembly time determined by the fuels inertia

$$\tau = R/4c_s \qquad c_s \approx 7 \quad 10^7 \quad cm/s \quad (\text{for} \quad \text{T} = 10 \text{ keV})$$

$$n_e = n_i = 2 \times \frac{\rho(\text{g/cc})}{2.5}\ 6.022 \quad 10^{23} cm^{-3} = 4.8\rho \quad 10^{23} cm^{-3}$$

$$n_e \tau = 1.5 \quad 10^{14} = \left(4.8\rho \quad 10^{23}\right)\left(R/4 \times 7.6 \quad 10^7\right)$$

For typical ICF conditions

$$\Rightarrow \rho R = 1.5 \ \ 10^{14} \times 4 \times 7\ 10^7 / 4.8 \quad 10^{23} \approx 0.3 \text{ g/cm}^2$$

Maximum released energy

CELIA

$$\mathcal{E}_{\text{fus}} = \varepsilon_{DT} N_f = \frac{M_f}{2m_i} \Phi_B \varepsilon_{DT} = 3.4 \times 10^5 \Phi_B M_f$$

Φ_B "burned fraction" of the fuel

M_f mass of fuel (in g)

$\mathcal{E}_{\text{fus}}$ fusion energy

ε_{DT} - energy released in one reaction (17.6 MeV)

A few mg of DT are sufficient to produce an energy of several 100 MJ

To satisfy Lawson's criterion ($\rho R > 0.3$ g/cm^2) at the density of solid D-T (ρ_{sol} = 0.25 g/cm^3) would require R = 1 cm. This implies a large explosion.

Burning fraction in ICF

We need to calculate the "burned fraction" of the fuel

$$R(t) = R_f - c_s t \qquad t_{\max} = R_f / c_s$$

$$N_{tot} = \frac{1}{2} n_i V_f \qquad n_D = n_T = n_i / 2 \qquad V_f = \frac{4\pi}{3} R_f^3$$

$$N_f = n_D n_T \langle \sigma \mathrm{v} \rangle \int_0^{t_{\max}} \frac{4\pi}{3} \left(R_f - c_s t\right)^3 dt =$$

$$= \frac{4\pi}{3} n_D n_T \langle \sigma \mathrm{v} \rangle \left[-\frac{1}{4c_s} \left(R_f - c_s t\right)^4 \right]_0^{t_{\max}} = \frac{4\pi}{3} n_D n_T \langle \sigma \mathrm{v} \rangle \frac{R_f^4}{4c_s}$$

$$= \frac{4\pi R_f^3}{3} n_D n_T \langle \sigma \mathrm{v} \rangle \frac{R_f}{4c_s} = n_D n_T \langle \sigma \mathrm{v} \rangle V_f \tau$$

$R(t) = R_f - c_s t$ $\qquad t=0$ $R=R_f$ $\qquad t=t_{max}$ $R=0$

Burning fraction in ICF

$$N_{ion} = \rho_f V_f / m_i = n_i V_f$$ total number of ions

$$N_{tot} = N_{ion} / 2$$ maximum number of reactions

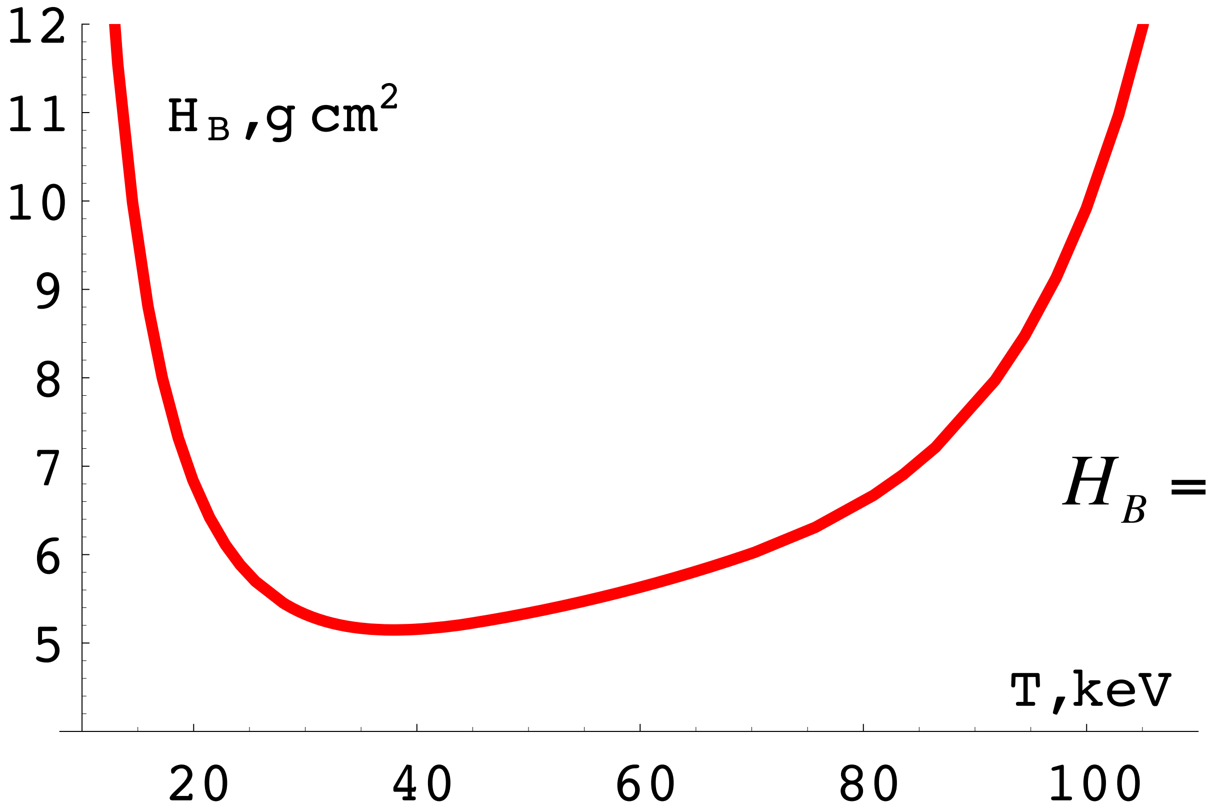
$$N_f = n_D n_T \langle \sigma \mathrm{v} \rangle V_f \tau = \frac{1}{4} n_i n_i \langle \sigma \mathrm{v} \rangle V_f \tau = \frac{1}{2} N_{tot} \langle \sigma \mathrm{v} \rangle n_i \tau$$

$$\Phi_B = \frac{N_f}{N_{tot}} = n_i \langle \sigma \mathrm{v} \rangle \frac{R_f}{8c_s} = n_i m_i \langle \sigma \mathrm{v} \rangle \frac{R_f}{8c_s m_i} = \frac{\rho_f R_f}{H_B}$$

$$H_B = \frac{8 m_i c_s}{\langle \sigma \mathrm{v} \rangle}$$

Burning fraction in ICF

H_B "combustion parameter" for a DT plasma fuel



$$H_B = \frac{8 m_i c_s}{\langle \sigma v \rangle} \approx 6 \quad g / cm^2$$

Burning fraction in ICF

The calculation didn't take into acount fuel consumption

$$n_f = n_{DO} - n_D$$

$$\frac{dn_f}{dt} = -\frac{dn_D}{dt} = n_D^2 \langle \sigma \mathrm{v} \rangle$$

$$H_B = \frac{8 m_i c_s}{\langle \sigma \mathrm{v} \rangle} \approx 6 \quad g / cm^2$$

$$n_D(t) = \frac{n_{DO}}{1 + n_{DO} \langle \sigma \mathrm{v} \rangle t}$$

$$t_{\max} = R_f / 4 c_s$$

$$n_{DO} = n_i / 2 = \rho_f / 2 m_i$$

Burning fraction in ICF

$$n_D(t_{\max}) = \frac{n_{DO}}{1 + n_{DO}\langle\sigma \mathrm{v}\rangle\left(R_f / 4c_s\right)}$$

$$\Phi = \frac{n_{DO} - n_D(t_{\max})}{n_{DO}} = 1 - \frac{n_D(t_{\max})}{n_{DO}} = 1 - \frac{1}{1 + n_{DO}\langle\sigma \mathrm{v}\rangle\left(R_f / 4c_s\right)}$$

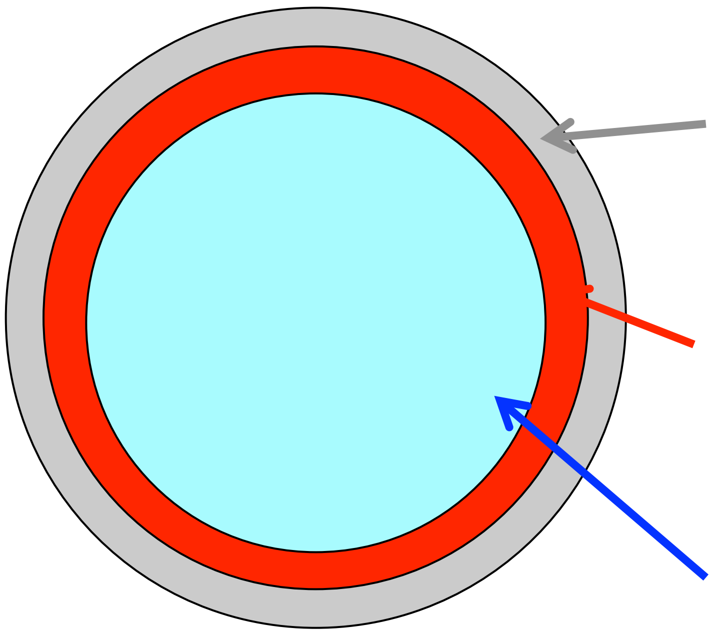
$$= 1 - \frac{1}{1_i + \left(\rho / 2m_i\right)\langle\sigma \mathrm{v}\rangle\left(R_f / 4c_s\right)} = 1 - \frac{1}{1 + \rho R_f / H_B}$$

$$= \frac{\left(1 + \rho R_f / H_B\right) - 1}{1 + \rho R_f / H_B} = \frac{\rho R_f / H_B}{1 + \rho R_f / H_B} = \frac{\rho R_f}{H_B + \rho R_f} \approx \frac{\rho R_f}{\rho R_f + 6}$$

Conventionally we take $\rho R \approx 3$ g/cm^2 that is 33% burned fuel

$$H_B = \frac{8 m_i c_s}{\langle\sigma \mathrm{v}\rangle}$$

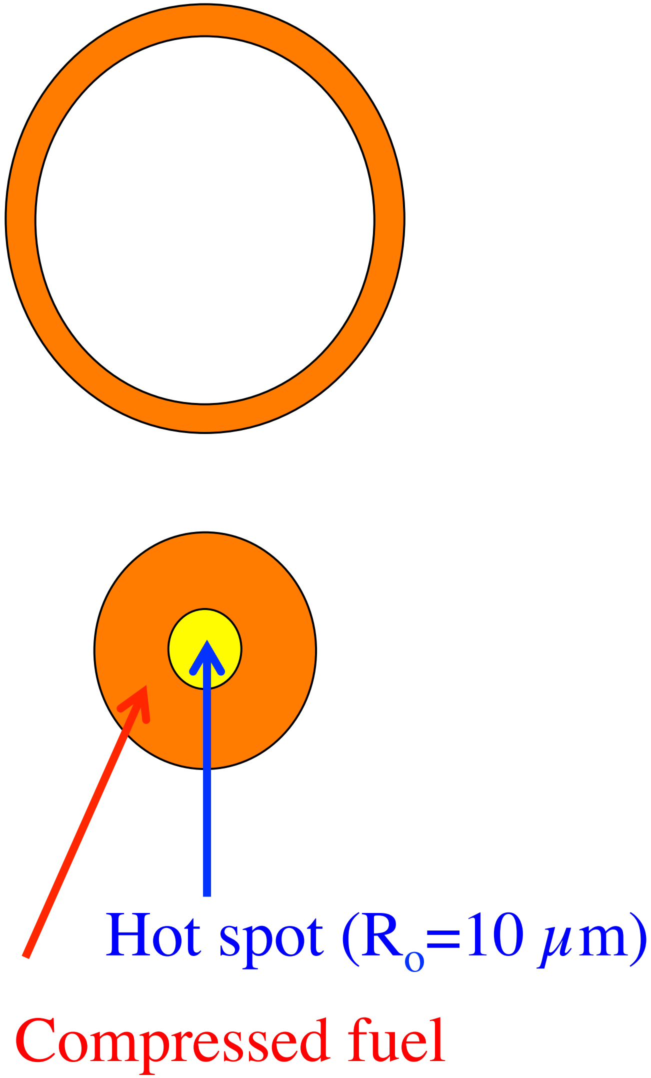
ICF typical targets



External radius ≈ mm

DT mass ≈ mg

ICF typical targets



INITIAL CONDITIONS

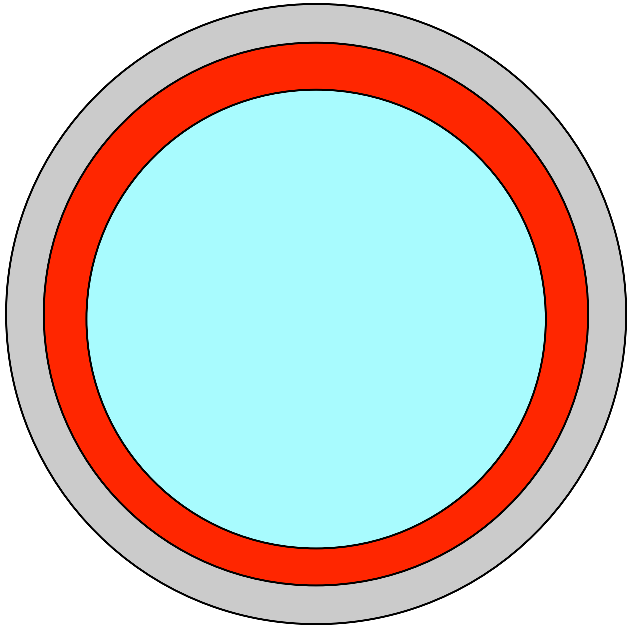
CRYOGENIC SHELL

- $R_{in} \approx$ 2 mm, $\Delta r \approx 33\ \mu$m
- $A = R_{in} / \Delta r \approx 60$
- $V_{in} \approx 4\pi R_{in}^2 \Delta r \approx 1.6\ \ 10^{-3}$ cm^3
- $\rho_{in} \approx 2.5 \times\ 0.1$ g/ cm^3, M $\approx$ 0.41 mg

FINAL CONDITIONS

- $\rho_{fin} / \rho_{in} \approx 1000$
- $\rho_{fin} \approx 250$ g/ cm^3
- $V_{fin} \approx 4/3\ \pi\ (R_{fin}{}^3 - R_o{}^3) \approx 1.6\ \ 10^{-6}$ cm^3
- $R_{fin} \approx 72\ \mu$m
- $\Delta r \approx 60\ \mu$m (A =1.2)
- $\rho_{fin}\ R_{fin} \approx 1.8$ g/ cm^2

Some orders of magnitude



If

$t_{laser} \approx t_{implosion} \approx 10$ ns

$I_L \approx 3\ 10^{14}$ W/cm^2

$R \approx 2$ mm

Then

$S = 0.5$ cm^2

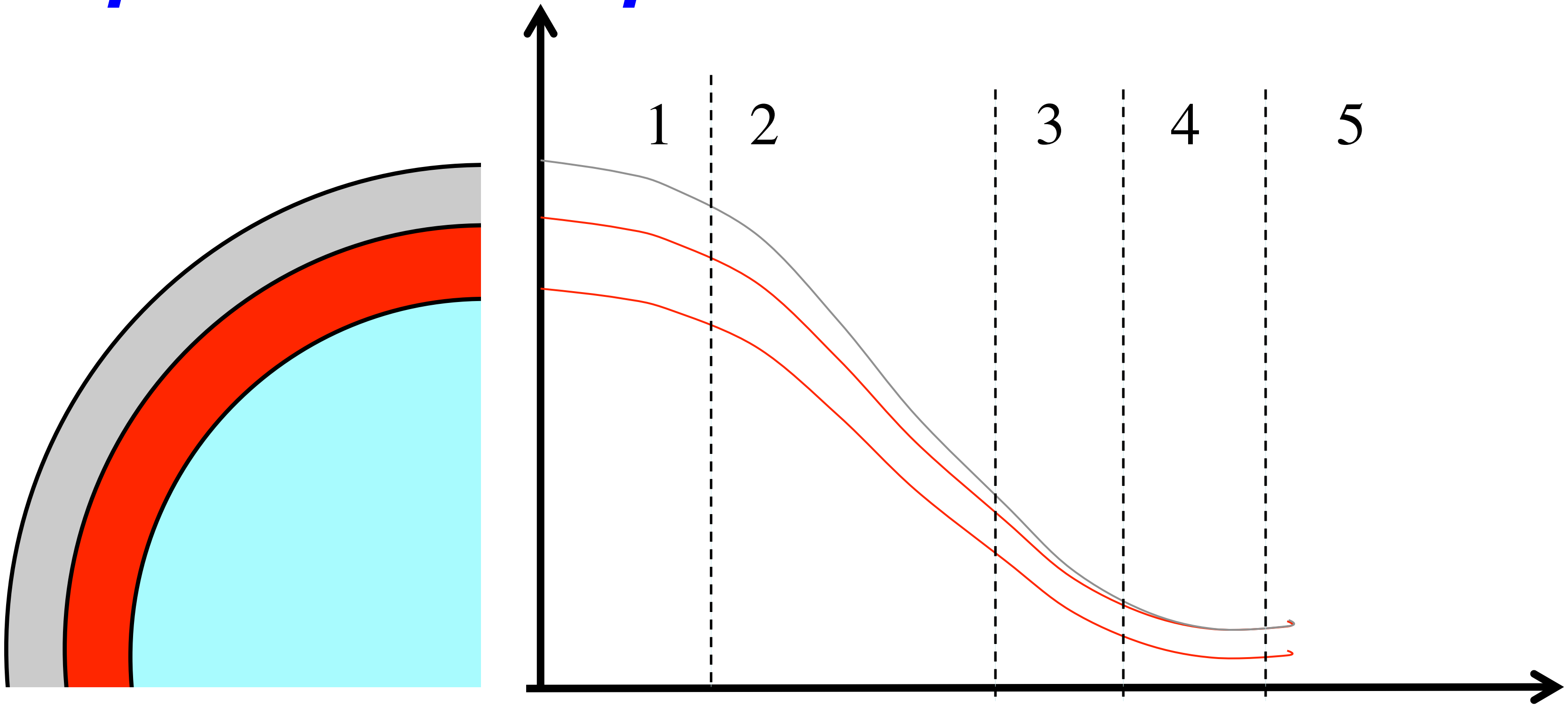
$E_{laser} = I_L t_{laser} S = 1.5$ MJ

$V_{implosion} = 2$ mm/10 ns

$= 200\ \mu$m/ns $= 200$ km/s

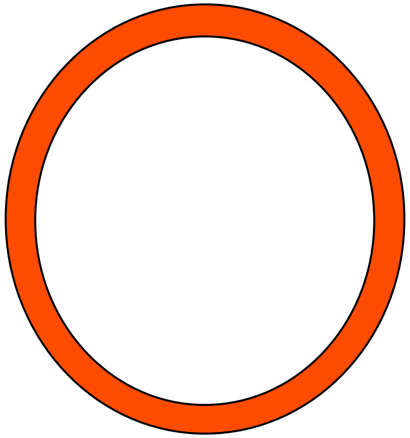
(in reality up to 400 μm/ns before "stagnation")

Space-time plot



1 ablation and acceleration
2 implosion (almost constant velocity)
3 deceleration
4 stagnation (creation of hot spot)
5 explosion

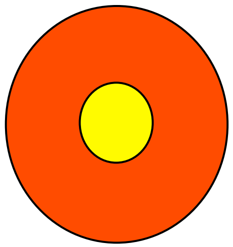
Why do we need a hot spot?



INITIAL CONDITIONS

CRYOGENIC SHELL

- $R_{in} \approx 2$ mm, $R_{in} / \Delta r \approx 60$ ($\Delta r \approx 33$ μm)
- $V \approx 4 \pi R_{in}^2 \Delta r \approx 1.6 \ 10^{-3}$ cm^3
- $\rho_{in} \approx 2.5 \times 0.1$ g/ cm^3, $M \approx 0.41$ mg
- $N_{moli} = 1.6 \ 10^{-3}$



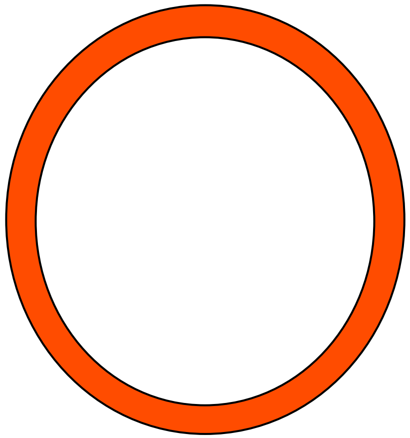
Total number of ions $N_{DT} \approx 2 \ 10^{21}$

If $T_{fin} \approx 10$ keV, total thermal energy in fuel $E \approx 2 (3/2 \ N_{DT} T) \approx 10$ MJ !!

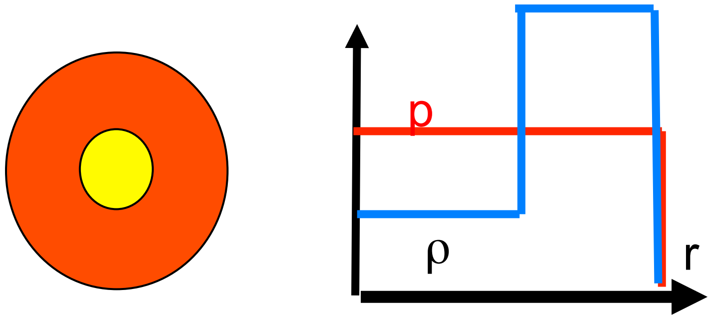
“Volume ignition” is NOT achievable !

Conversion efficiency from laser light to thermal energy of the fuel is extremely low ≈ 5 %

"Isobaric" approach to ICF



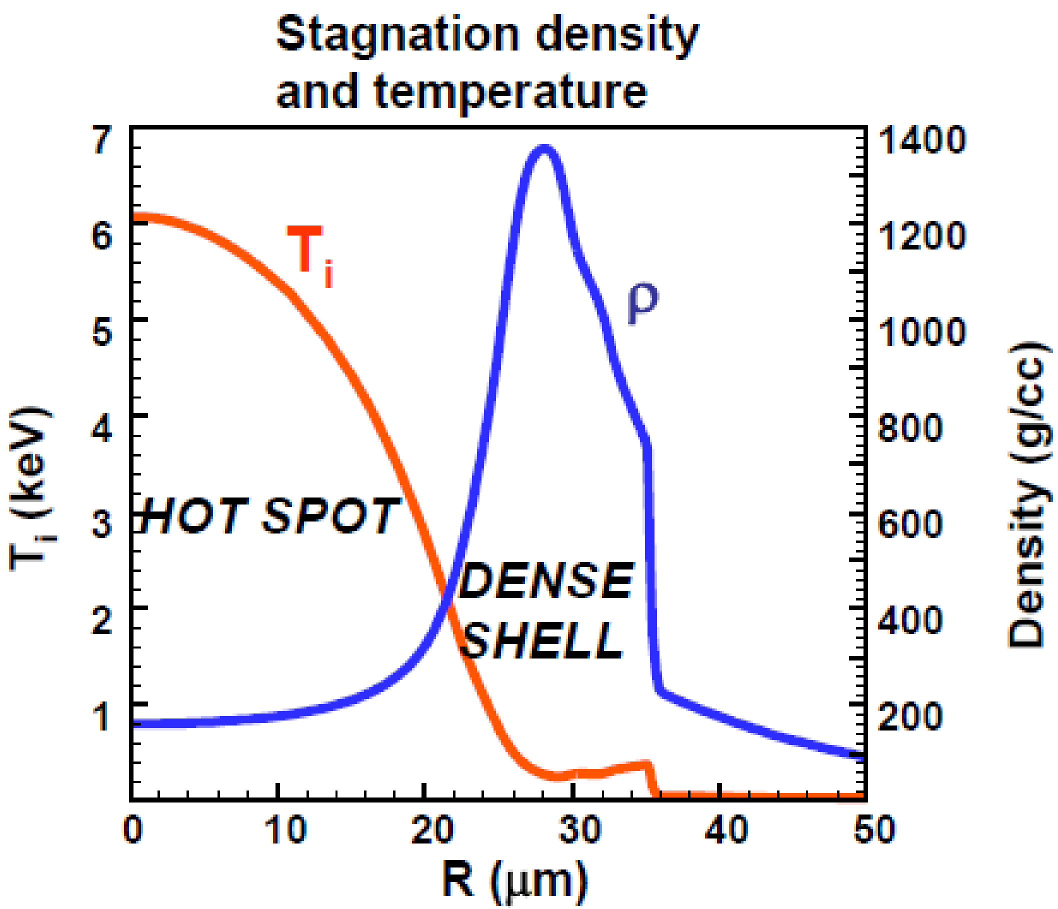
Stagnation is reached as the pressure of the internal fuel increases and gradually slows the shell down.
At stagnation $P_{shell} \approx P_{central\ spot}$



Kinetic Energy of (remaining) imploding shell is converted in:
- Compression of the fuel in the shell
- Heating of the central has

Produces an Isobaric fuel assembly
Need for High Aspect ratio target and high implosion velocities
V ~400 km/s

NIF-like target (1 MJ)



$$n_e = n_i \; = 2.4\rho \quad 10^{23} cm^{-3}$$

$$P(Bar) = 1.8 \;\; 10^{-18} \quad n_{TOT}(cm^{-3}) \; T_e(eV)$$

SHELL

-$\rho_{fin} R_{fin} \approx 10^3$ g/ cm$^3 \times 40\ \mu$m

= 4 g/ cm^2

-$n_i = n_e \approx 2.5\ 10^{26}$ cm^{-3}

HOT SPOT

-$\rho_{fin} R_{fin} \approx 130$ g/ cm$^3 \times 22\ \mu$m

= 0.3 g/ cm^2

-$n_i = n_e \approx 3\ 10^{25}$ cm^{-3}

PRESSURE (hot spot)

-P $\approx$ 600 GBar

Spherical geometry

Notice:

Ablation Pressure P $\approx$ 50 MBar

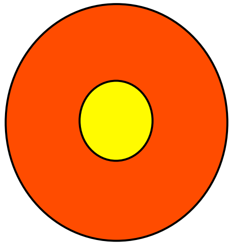
Pressure at stagnation P $\approx$ 500 Gbar

Amplification of a factor $\times$ 10000 due to convergence.

Spherical geometry is essential for ignition

Energy balance in the fuel

The compressed shell is a dense degenerate plasma



$$T_F = (3\pi^2 n)^{2/3}\hbar^2/2m,$$

$$T_F \simeq 14\,\rho^{2/3}\ \mathrm{eV},$$

$$p_F = \tfrac{2}{5} n_e T_F.$$

$$p_F = A_F \rho^{5/3} = 2.16\,\rho^{5/3}\mathrm{Mbar}.$$

“entropy parameter”

$\alpha = p/p_F$ $\qquad$ $p = \alpha\, A_F\, \rho^{5/3}$

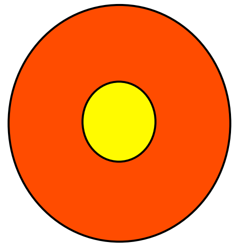
The hot spot is a hot classical plasma

Energy balance in the fuel

The compressed shell is a dense degenerate plasma

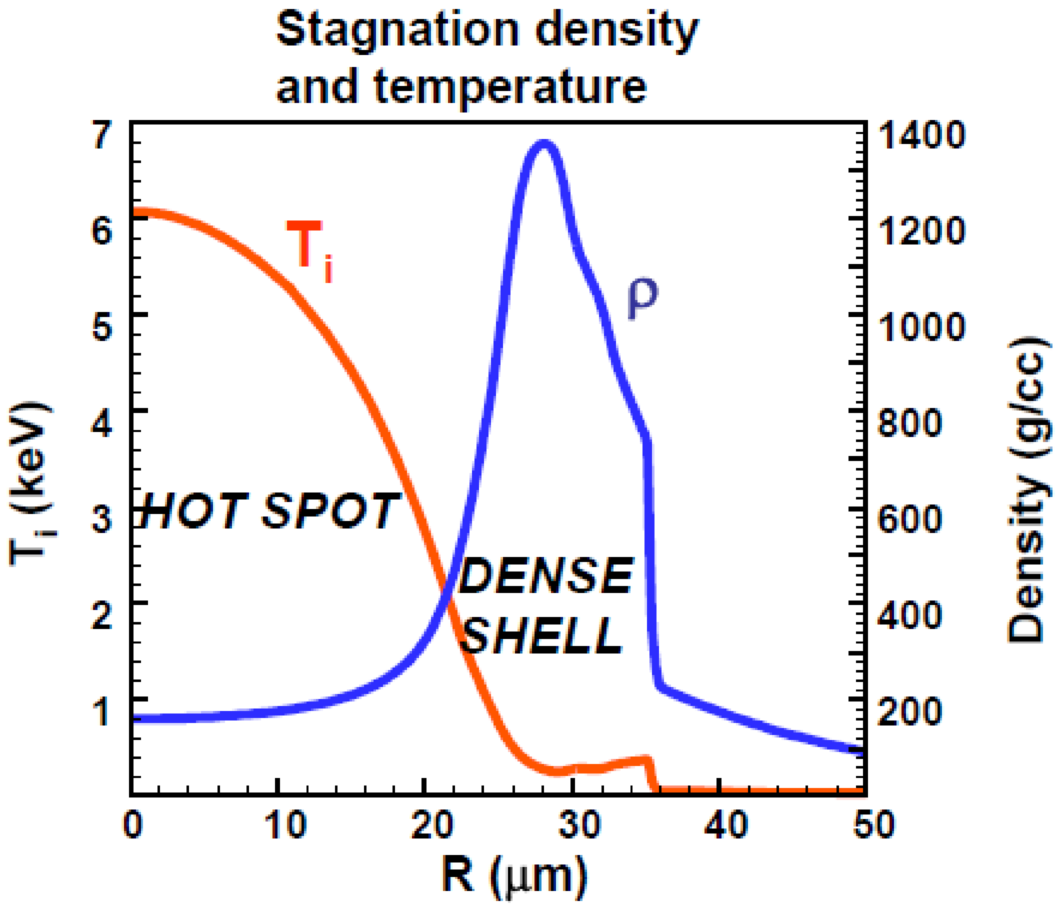
$$\mathcal{E}_{\text{compr}} = \int_{V_f}^{V_0} p\, dV = \alpha M_f \int_{\rho_0}^{\rho_f} \frac{p_F}{\rho^2}\, d\rho \simeq \frac{3}{2}\alpha M_f A_F \rho_f^{2/3} = 0.35\, \alpha\, M_f \rho_f^{2/3}\, \text{MJ}.$$

The hot spot is a hot classical plasma



$$\mathcal{E}_{\text{chauf}} = \frac{3}{2}\,(n_e + n_D + n_T)\, T_h V_h = 3\, n_e T_h V_h = 110\, M_h T_{\text{keV}}\, \text{MJ}.$$

NIF-like target (1 MJ)



SHELL

-$\rho_{fin} R_{fin} \approx 10^3$ g/ cm$^3 \times 40\ \mu$m

= 4 g/ cm^2

-$n_i = n_e \approx 2.5\ 10^{26}$ cm^{-3}

HOT SPOT

-$\rho_{fin} R_{fin} \approx 130$ g/ cm$^3 \times 22\ \mu$m

= 0.3 g/ cm^2

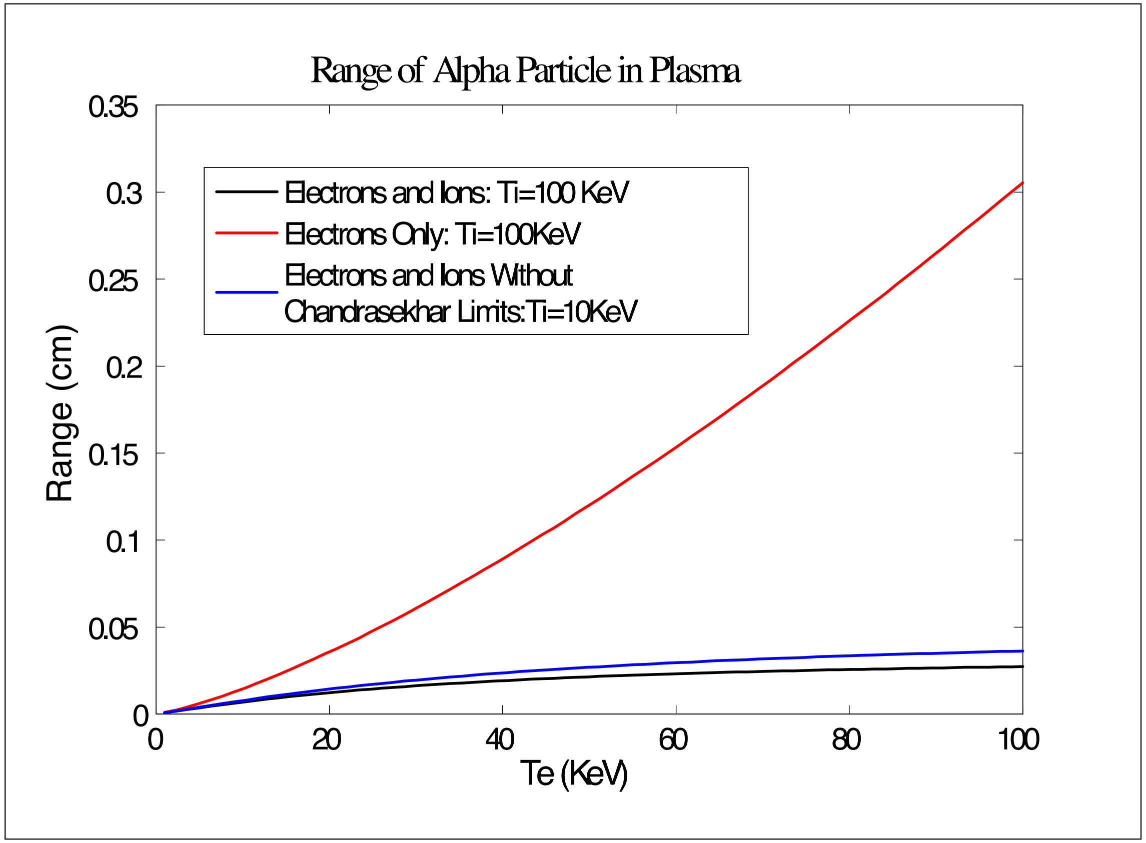
-$n_i = n_e \approx 3\ 10^{25}$ cm^{-3}

PRESSURE (hot spot)

-P $\approx$ 600 GBar

$$p_F = \alpha A_F \rho^{5/3} = 2.16\ \alpha\ \rho^{5/3}\ \ MBar$$
$$= 216\ \alpha\ \ Gbar \qquad \Rightarrow \alpha \approx 3$$

α particle range in ICF plasma



$$R_a = 0.107 \frac{T_e^{3/2}}{\rho \ln(\Lambda)} [cm]$$

Assuming 50/50 DT plasma (T_e in keV, ρ in g/cm^3). The plot of the range is shown in figure for ρ = 50 g/cm^3 and T_i= 100 keV

To get $R_\alpha \approx 10\ \mu m$, one needs $\rho \approx 500$ g/cm^3 at T ≈ 5 keV

In this case α particles are confined within the plasma and contribute to its heating

Lessons from History ...

CELIA

Laser Compression of Matter to Super-High Densities: Thermonuclear (CTR) Applications

JOHN NUCKOLLS, LOWELL WOOD, ALBERT THIESSEN & GEORGE ZIMMERMAN
University of California Lawrence Livermore Laboratory

Nature **239**, 139 - 142 (15 September **1972)**

Hydrogen may be compressed to more than 10,000 times liquid density by an implosion system energized by a high energy laser. This scheme makes possible efficient thermonuclear burn of small pellets of heavy hydrogen isotopes, and makes feasible fusion power reactors using practical lasers.

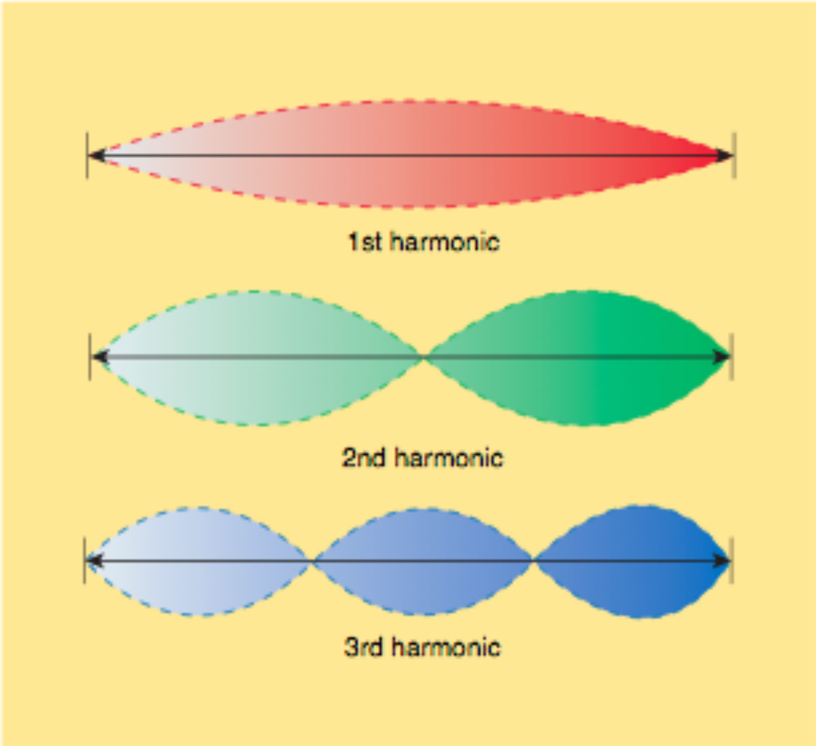
Early predictions for laser ignition were ~1kJ [Nuckolls]

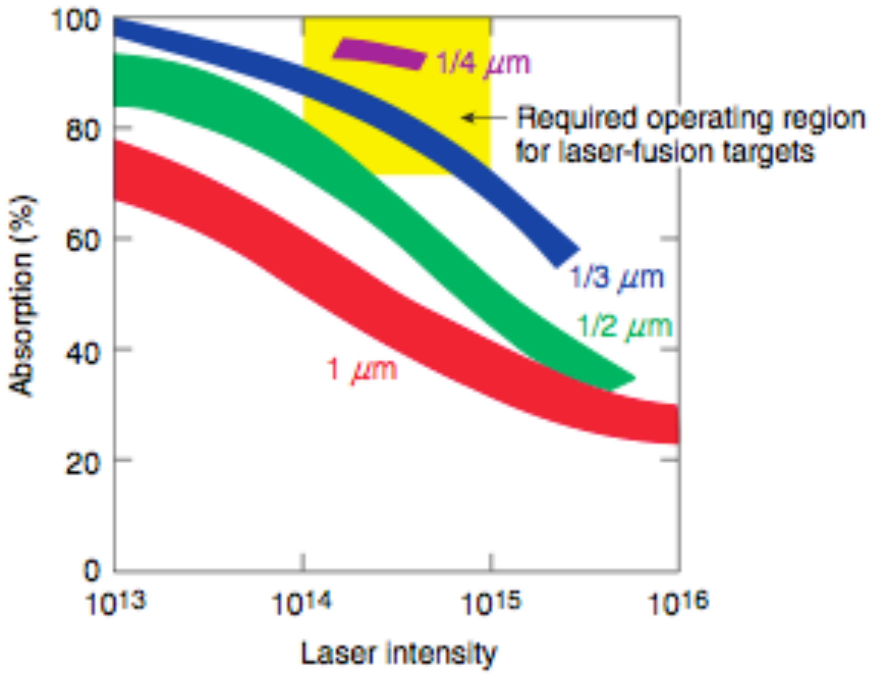
- **What was wrong?**
  - Very strong sensitivity to implosion velocity
  - Optimistic assessment of hydro instability growth
  - Assumption of high coupling efficiency at high laser irradiance

Note: estimates for high gain have remained ~ constant (at ~MJ), as much weaker dependence on implosion velocity

Absorption ruled out IR lasers

Laser Matter Interaction prohibits large intensities (low absorption, high preheating)





Why we cannot work at high intensity and high wavelength

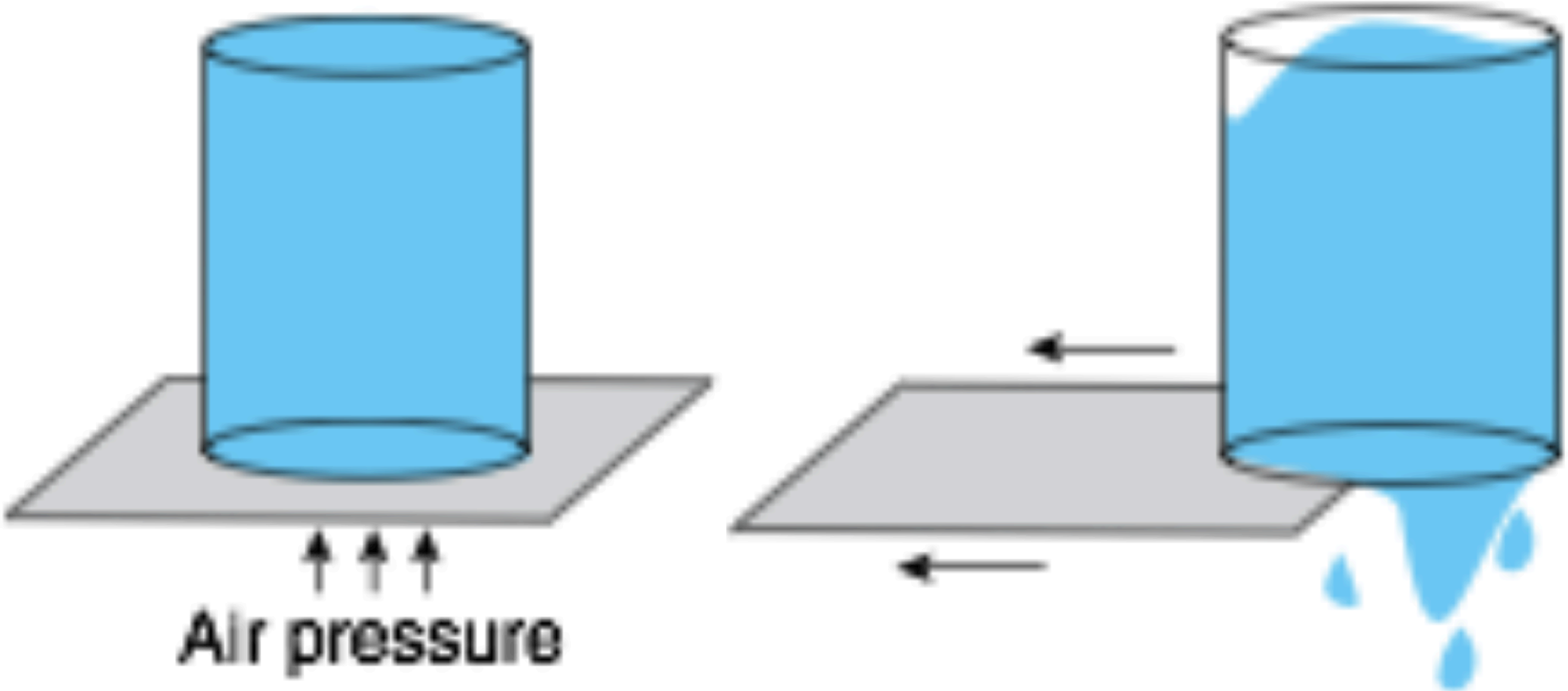
1) Absorption is reduced

2) Plasma corona is hotter, $T_e \approx (I_L \lambda^2)^{2/3}$ producing more and harder X-rays

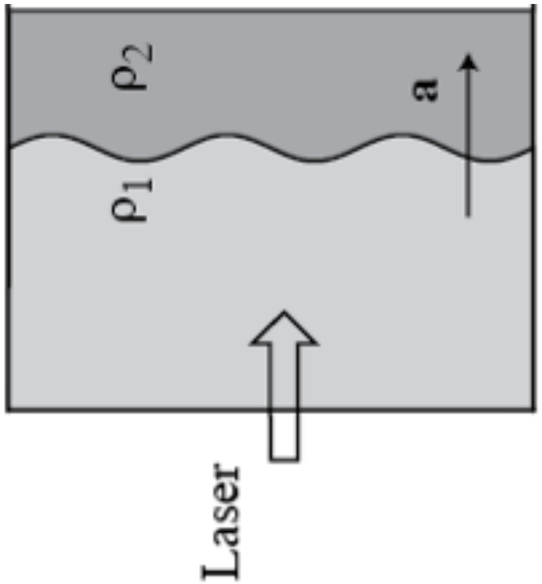
3) Parametric instabilities are excited more easily (SRS, TPD) and may produce hot electrons (non thermal tail in electron distribution function)

4) Hot electrons and X-rays may "preheat" the fuel making its compression much more difficult

Rayleigh-Taylor Instability:



Rayleigh-Taylor Instability:



Use Fourier expansion (we write a sinusoidal perturbation).
Here $A(x,t) = A_O \cos(kx - \omega t)$

The equation of dynamics bring to write a dispersion relation $\varepsilon(\omega,t) = 0$

If ω is real stable behaviour (wave which is eventually damped and dies away)
If $\omega = i\gamma$ is imaginary then the perturbation grows like
$A(x,t) = A_O \cos(kx)\, e^{\gamma t}$

Rayleigh-Taylor Instability:

"Classical" growth rate (linear phase)

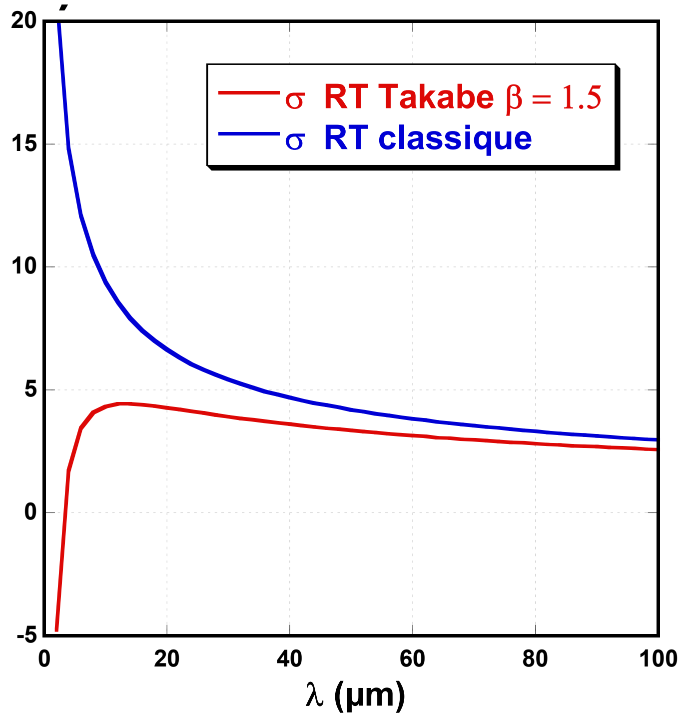
$$\gamma = \sqrt{Akg}$$

$$A = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1}$$

Atwood number (unstable if $\rho_1 < \rho_2$)

$$\gamma = \sqrt{\frac{Akg}{1+kL}} - \beta k v_{abl}$$

γ (ns^{-1})



Modified "Takabe" expression. L plasma density gradient

Rayleigh-Taylor Instability:

In ICF, Rayleigh Taylor instability is partially stabilised due to

- the presence of a density gradient ($\rho(x)=\rho_o \exp(-x/L)$) (less space for growth)
- plasma expansion (the "seeded instability" is transported away)

$$\gamma = \sqrt{\frac{Akg}{1+kL}} - \beta k v_{abl}$$

- Short modes are stabilised ($\lambda << L$, i.e. $kL >> 1$)
- Large modes are not stabilised but they grow very slowly (λ is big and then k and γ are small)

The most dangerous modes are the intermediate ones

Lessons from History ...

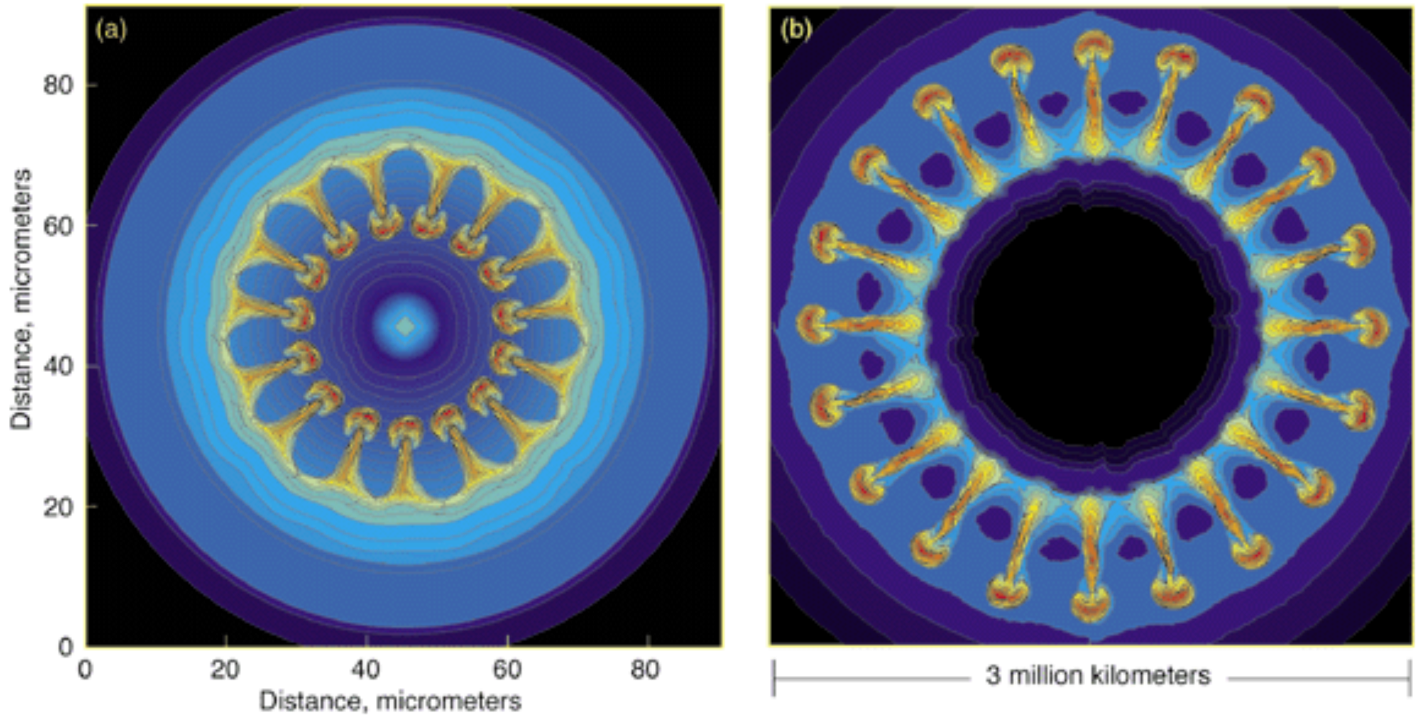
In conclusion, what was wrong in the initial energy estimate from the 70's? [~1kJ , Nuckolls]

They wanted to use long wavelength lasers (CO_2, Nd working at 1ω) because they are very efficient but this implies LOW ABSORPTION, and HOT PLASMA CORONA inducing X-rays and PREHEATING

They neglected the impact of parametric instabilities (especially effective at high intensity and long wavelength) creating PLASMA WAVES inducing HOT ELECTRONS and PREHEATING

In order to get strong accelerations, they wanted to use high intensities to create big pressure and low mass targets (thin shells) but in this case hydro instabilities are very effective

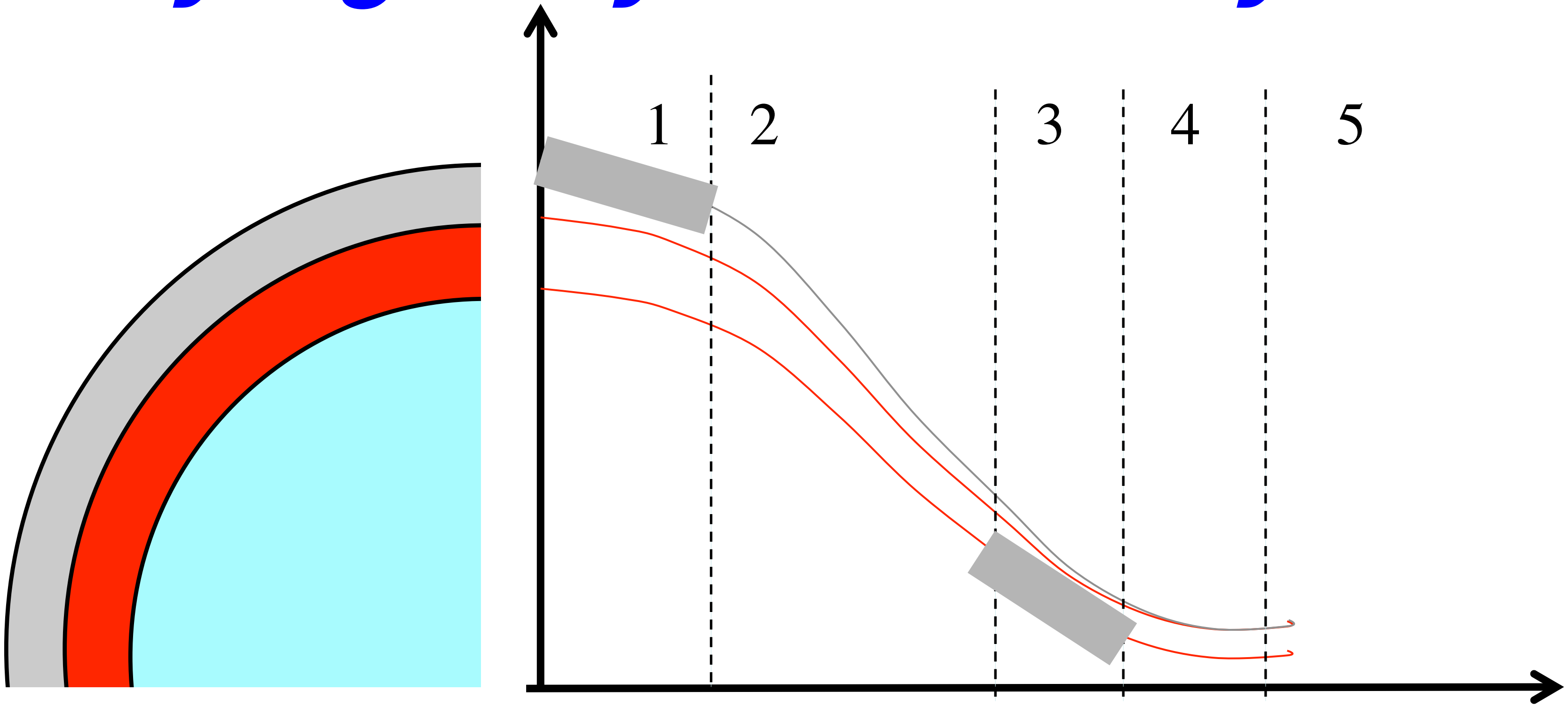
Rayleigh-Taylor Instability – spherical implosions / explosions



Striking similarities exist between hydrodynamic instabilities in (a) inertial confinement fusion capsule implosions and (b) core-collapse supernova explosions. [Image (a) is from Sakagami and Nishihara, *Physics of Fluids B* **2**, 2715 (1990); image (b) is from Hachisu et al., *Astrophysical Journal* **368**, L27 (1991).]

- **Energy must be delivered as sysmmetric as possible!**

Rayleigh-Taylor Instability:



In ICF Rayleigh Taylor instabilities may develop:
1) during the acceleration phase (1) at the ablation front [the less dense plasma corona "pushes" the denser shell]
2) During the deceleration phase (3) at the shell/fuel inner interface [the less dense gas in the core pushes the imploding shell]

Rayleigh-Taylor Instability:

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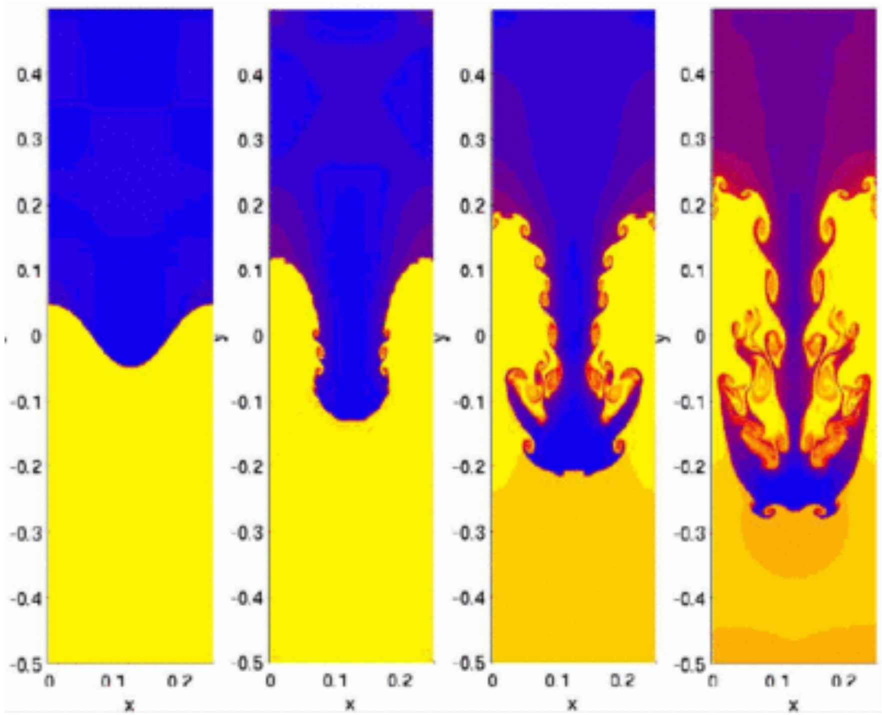
RT instability is dangerous for ICF because:

1) It may bring to mixing of shell materials (C, H, Si, O, …) with fuel. This increases the average Z of the core bringing to increased radiation emission and losses
2) In the most severe case, it may "break" the shell thereby terminating implosion

In order to achieve the high implosion velocities needed for ignition ($V_{imp} \approx 400$ km/s) one needs to accelerate a small mass, i.e. a thin shell (high aspect ratio). But of course a thin shell is more sensitive to hydro instabilities…

Rayleigh-Taylor Instability

- Major instability: heavy material pushes on low density one
- Will always occur since driver is never 100% symmetric
- The Rayleigh-Taylor instability always grows



➤**Energy must be delivered as sysmmetric as possible!**

Good news:

Experiments with GEKKO XII laser, Institute of Laser Engineering, University of Osaka, Japan

- **Experimental demonstration of compression of DT up to 600 × solid density** (Azechi et al., Las. Part. Beams, 1991)

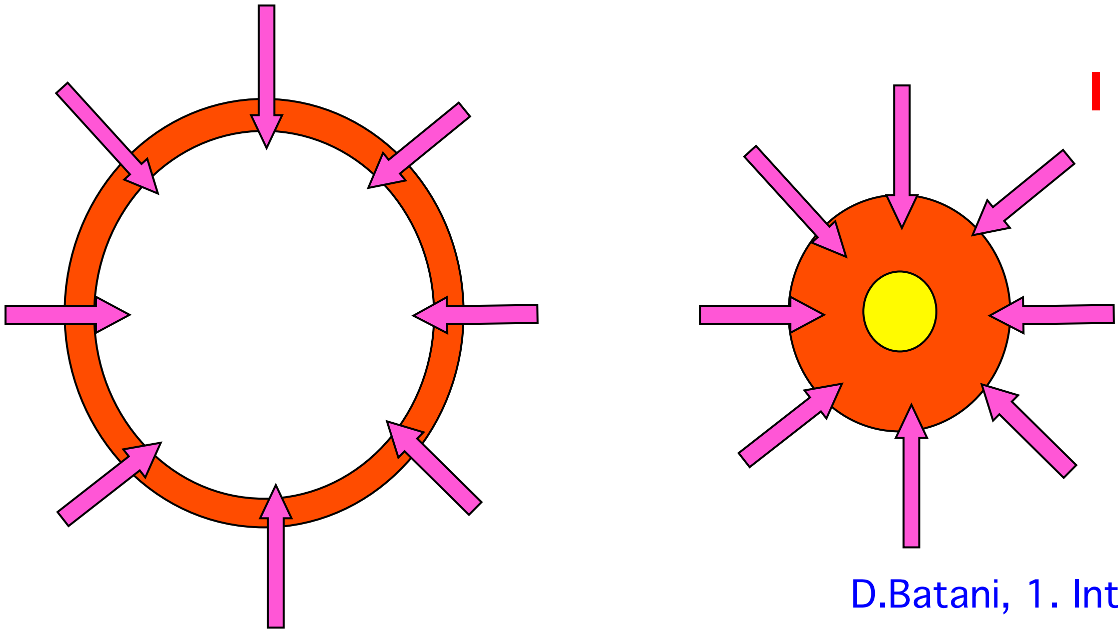
- **WE ARE ABLE TO OBTAIN DENSITIES DIRECTLY RELEVANT FOR ICF!**

  However: number of neutrons much smaller than expected: The central hot spot was not generated

ICF: direct drive approach

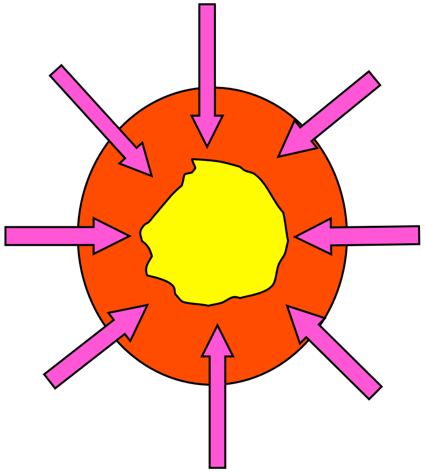
Lawson's Criterium (buring criterum) for ignition (D-T): $\rho R > 3$ gcm^{-2}
and $T \approx 5$ - 10 keV

- synchronized laser pulses with spherical irradiation symmetry
- shock wave compression (up to 1000 × solid density)
- ignition from a central hot spark produced by shock coalescence
- isobaric approach



$I \approx 10^{14}$ W/cm^2 10 ns

Problems of classical scheme:



Non uniformities in laser irradiation or in target bring to:

- **Mixing of fuel and wall, higher Z*, increased emission and cooling**
- **The central hot spot is not generated**

$$\frac{\Delta R_{fin}}{R_{fin}} \approx \frac{\Delta v}{R_{fin}} t_{imp} \approx \frac{R_{in}}{R_{fin}} \frac{\Delta v}{v_{imp}} \approx \frac{R_{in}}{R_{fin}} \frac{\Delta I}{I}$$

Then:

$$\frac{\Delta R_{fin}}{R_{fin}} \approx 50\% \Rightarrow \frac{\Delta I}{I} \approx 1\%$$

How to relax uniformity constraints for ICF?

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- **USE INDIRECT DRIVE**

- **USE OPTICAL SMOOTHING**

- **USE OF "FOAM BUFFERED TARGETS"**

Willi et al., Phys. Rev.Lett.,1995

- **SEPARATE COMPRESSION AND IGNITION**

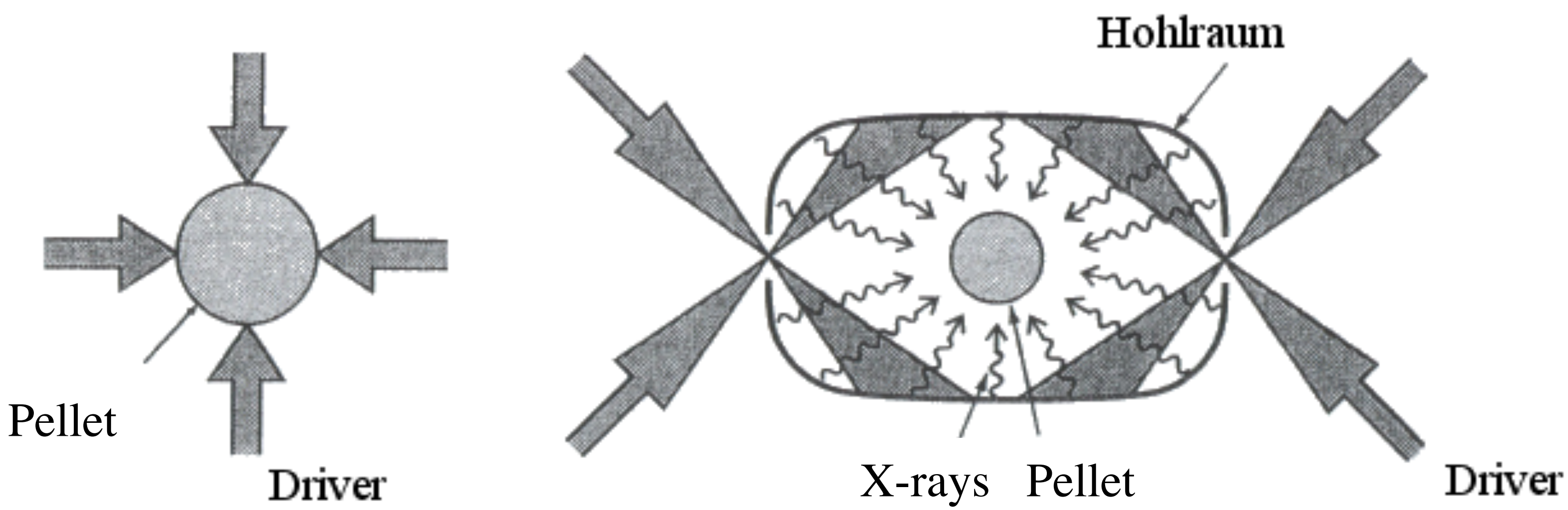
Fast Ignition

Tabak et al., Phys. Plasmas, 1994

Shock Ignition

R. Betti, et al. Phys. Rev. Lett., 2007

Inertial confinement: direct vs. indirect drive



Direct drive

Indirect drive

Direct: higher efficiency, more problems with uniformity

Indirect: better uniformity but reduction of efficiency

In both case you need MJ-class laser systems

Targets

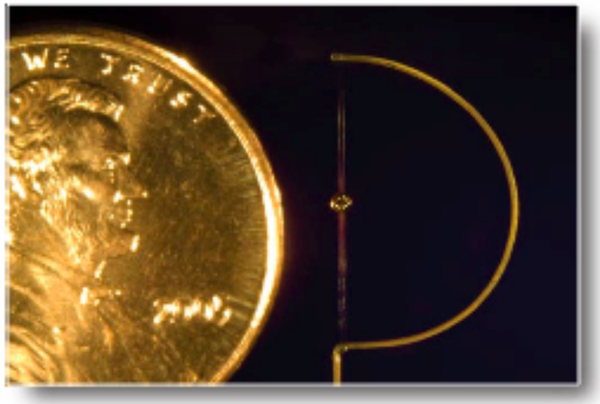


Figure 3.3: The laser-fusion targets used on OMEGA experiments are typically ~1 mm in diameter and are suspended by spider silks.

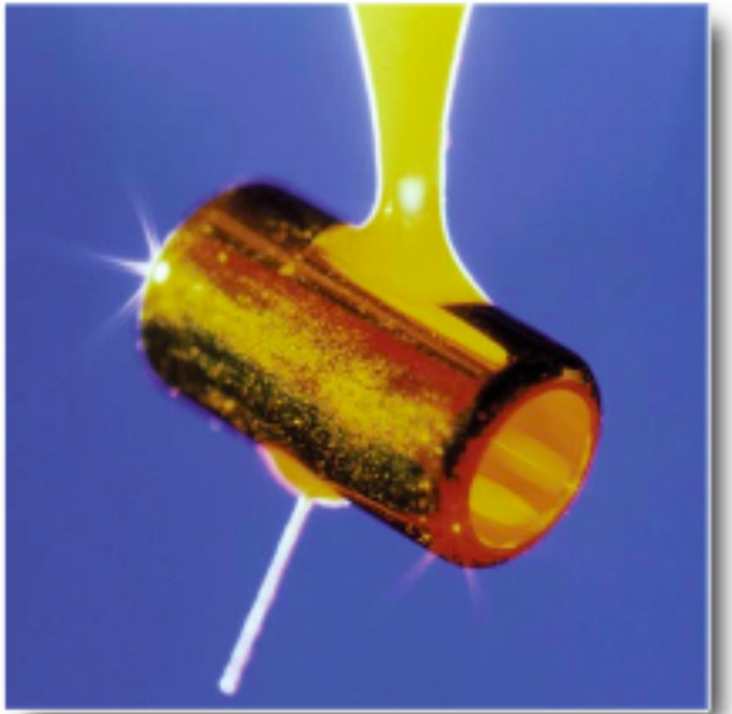
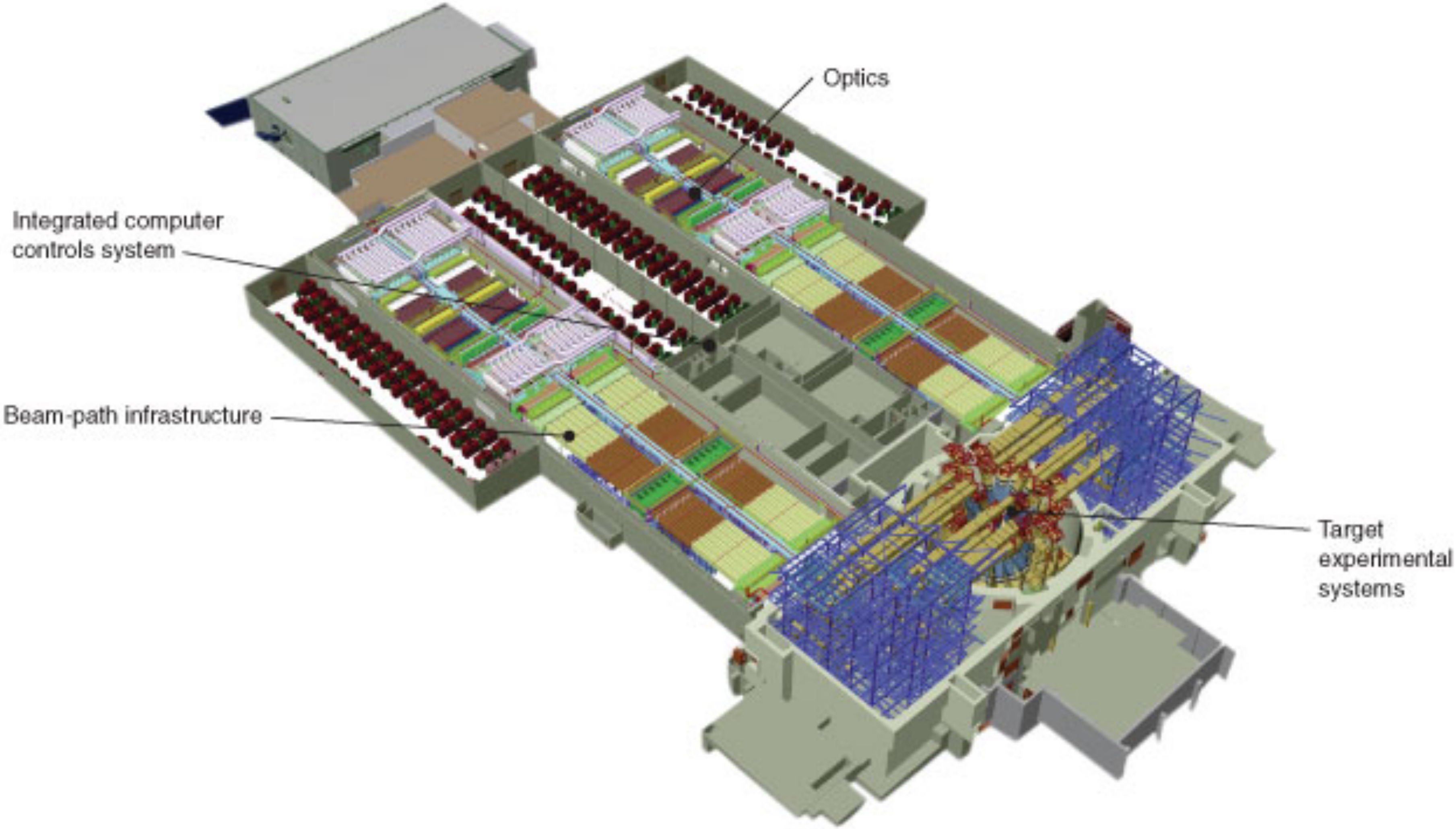
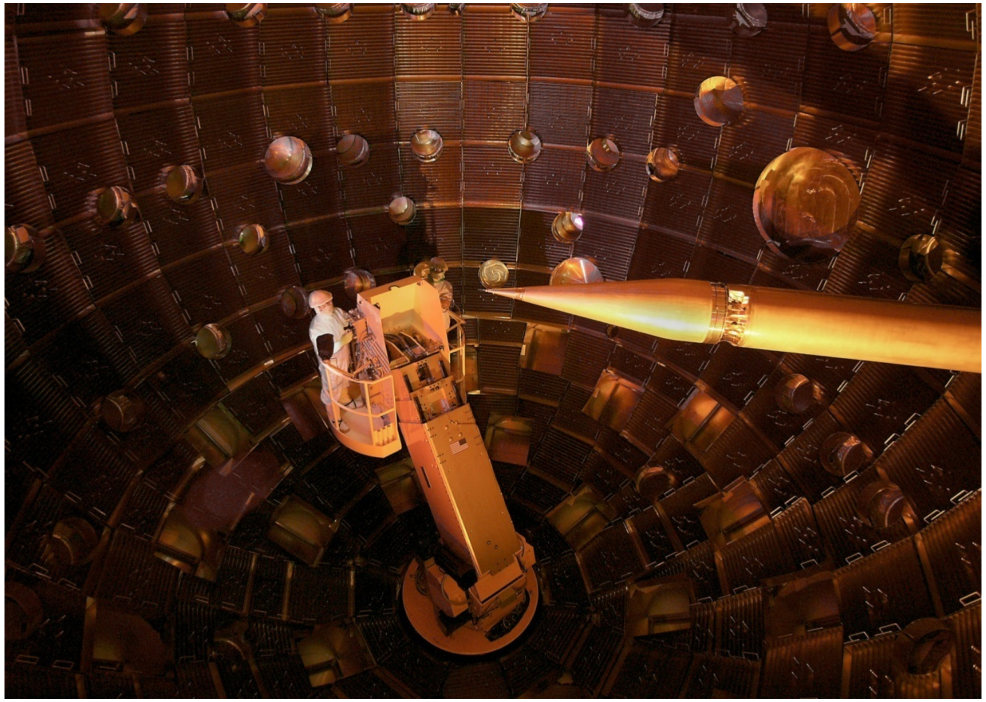


Figure 3.7: A typical indirect-drive target.

National Ignition Facility (NIF) - layout

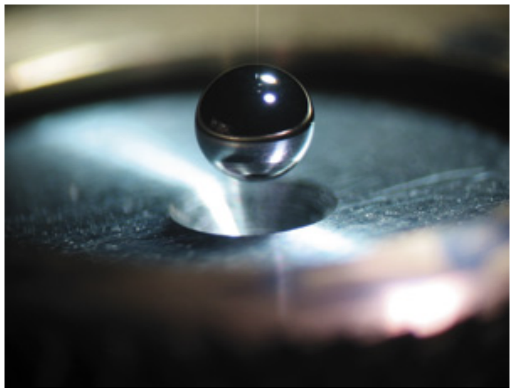


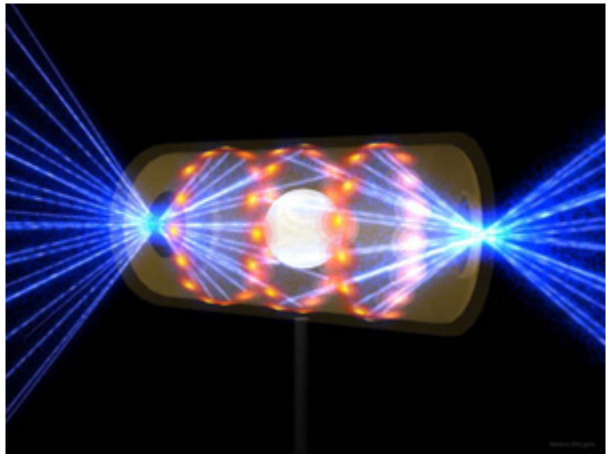
NIF



Indirect drive

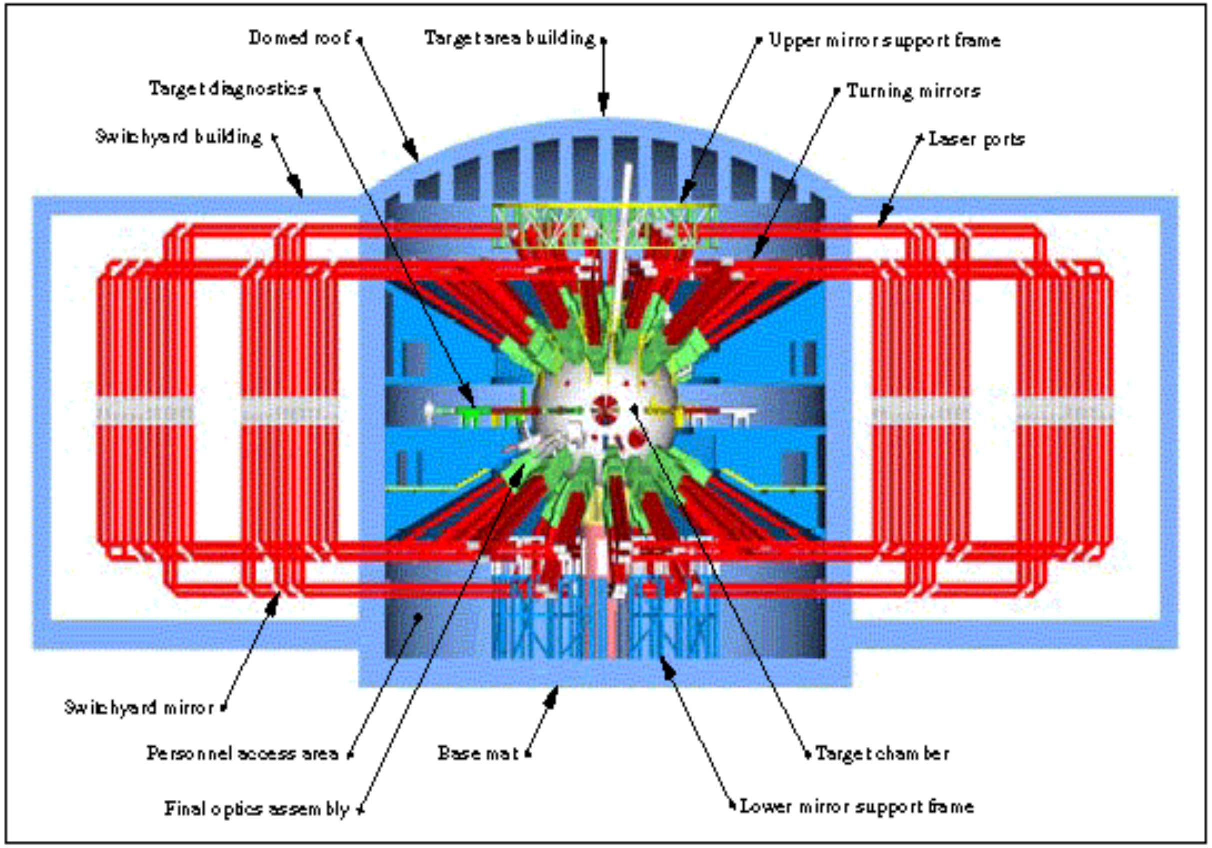
- Chamber
- Target holder
- Hohlraum
- Pellet





Lasers - NIF and Megajoule:

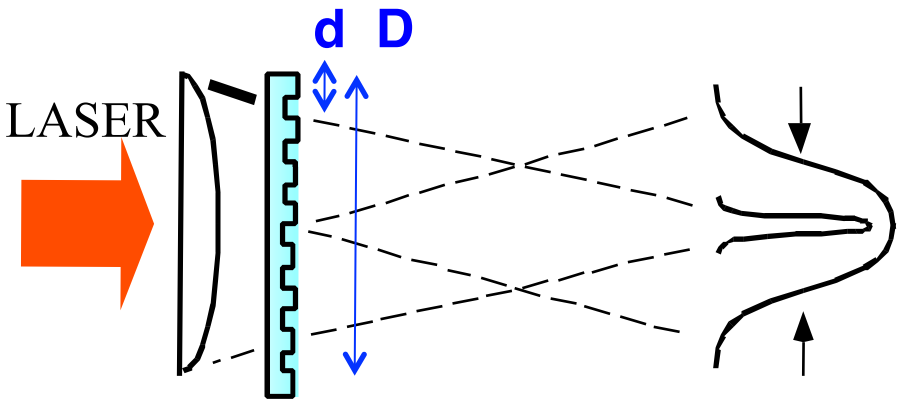
Controlled Thermonuclear Fusion

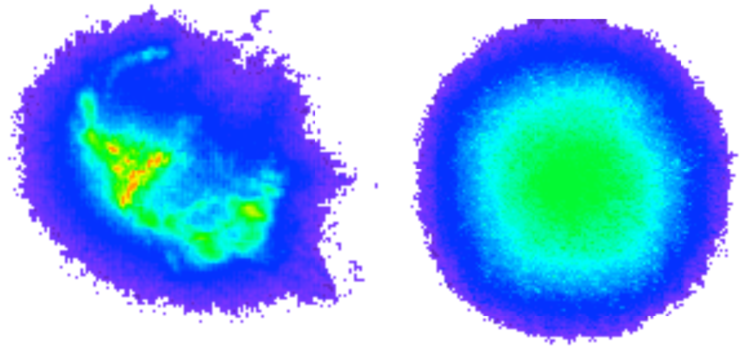


Nd:glass
2 MJ
10 ns
200 beams

Optical smoothing techniques:

Optical smoothing techniques (RPP, ISI, SSD..) introduced (80's, 90's) to produce "gaussian" beam profiles with small scale modulations





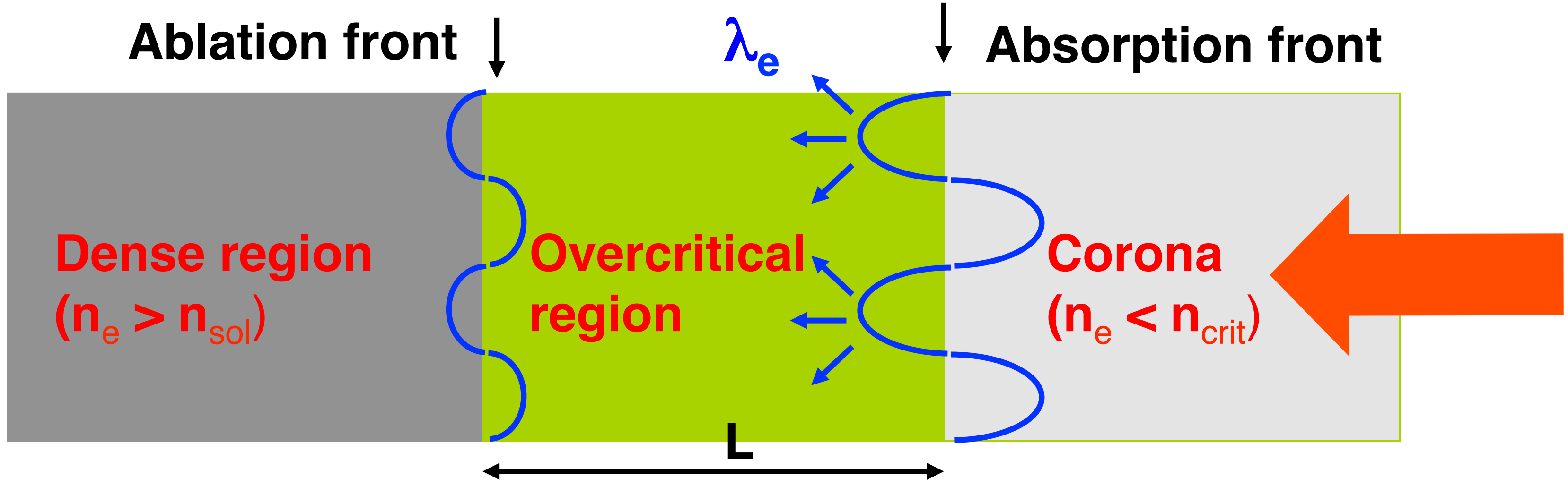
Random Phase Plate **Spot $\Phi \approx \lambda f/d$** **speckles $d \approx \lambda f/D$**

2D square elements with 0 or π dephasing *(Kato, PRL, 1984)*

Small scale modulations are rapidly washed out by thermal smoothing

Thermal smoothing:

Laser is absorbed at the absorption front but pressure is applied at the ablation front



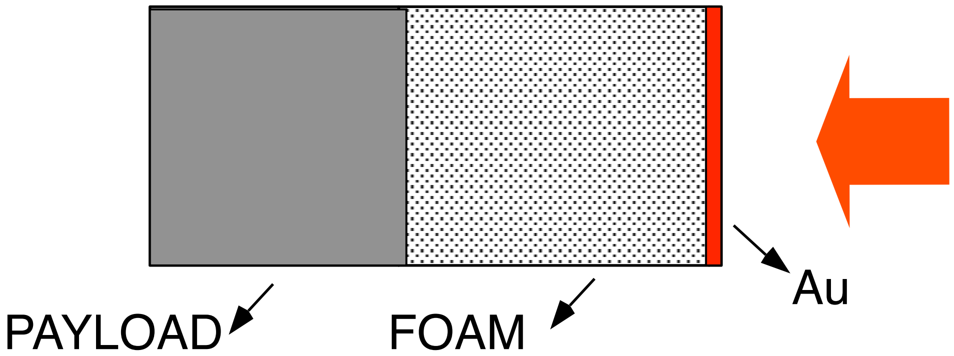
$$\Delta P / P \approx \Gamma \, \Delta I / I$$

Non-uniformities present at the laser absorption surface are reduced at the ablation surface by

$$\Gamma \approx \exp [- \alpha L (2\pi / \lambda_{perp})]$$

L "stand-off distance" (thickness of conduction zone)

Foam buffered targets:



(Dunne et al., PRL 1995)

CREATE A SUPERCRITICAL AND SUFFICIENTLY WIDE PLASMA IN FRONT OF THE TARGET

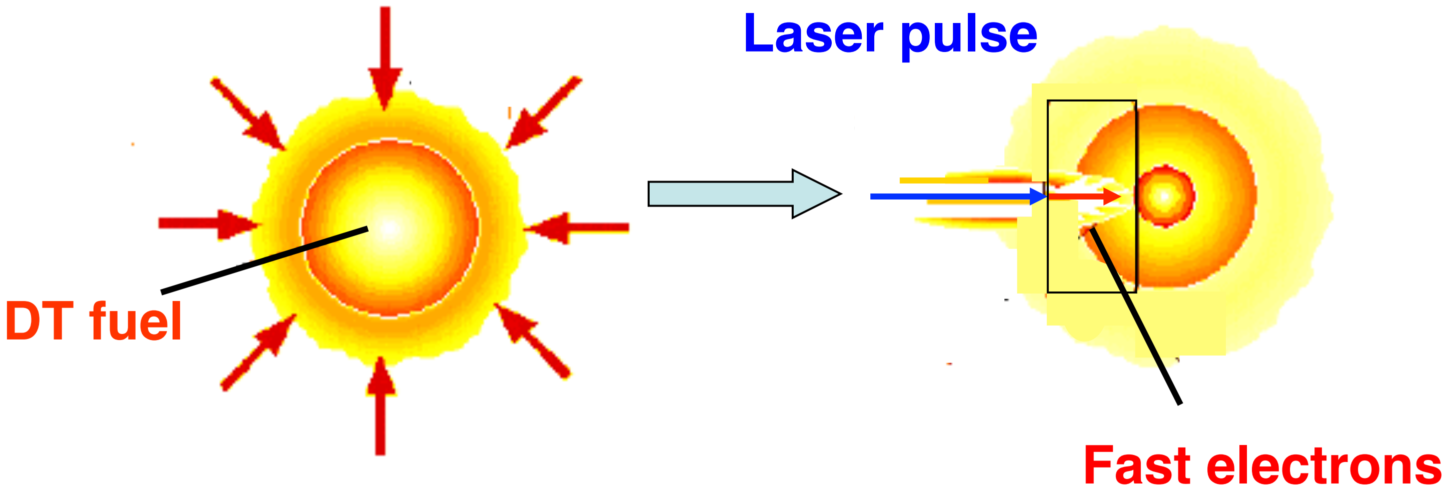
OPEN QUESTIONS:

- **Optimal parameters of foam and converter**
- **What λ_{perp}?**
- **Effects on hydrodynamics? Shock heating of DT?**

The concept of"fast ignition"

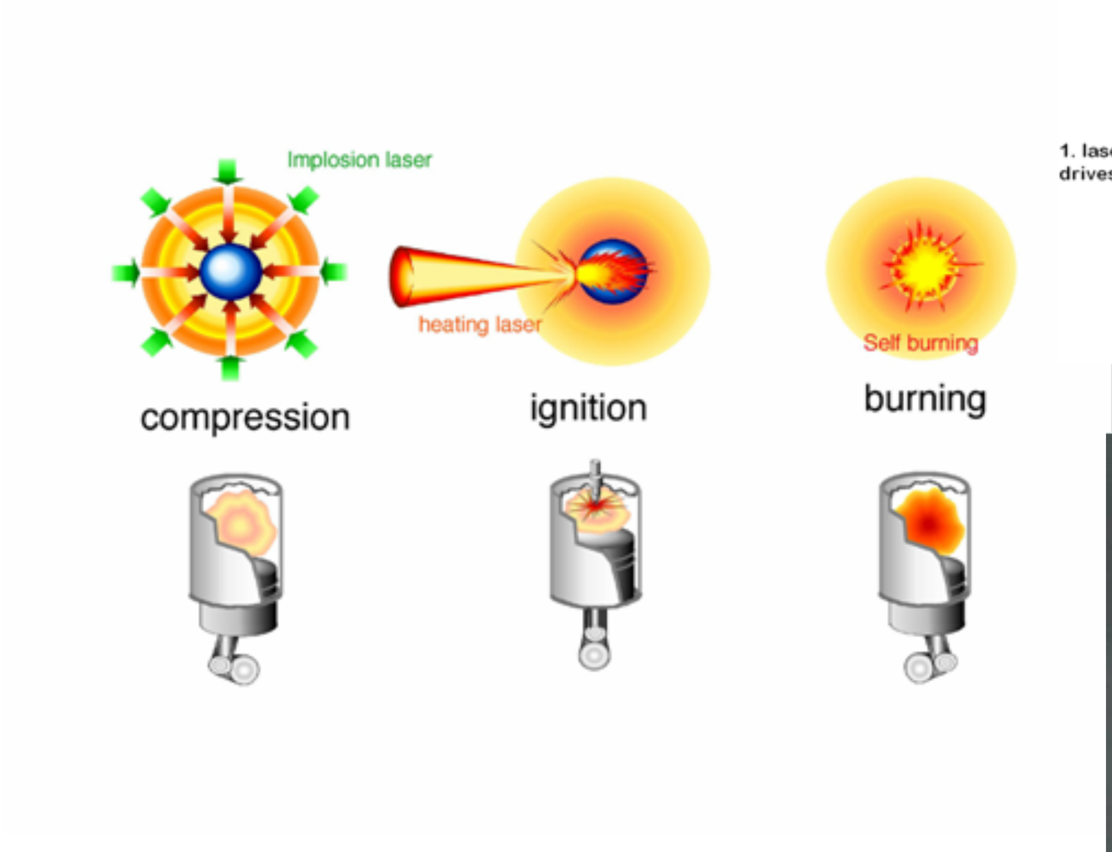
➢1: "normal" compression with ns laser beams **(we are able to compress!)**

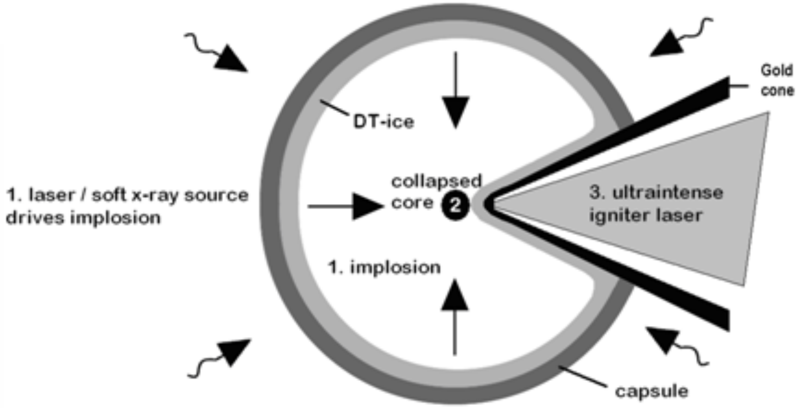
➢2: a CPA laser creates a beam of relativistic electrons (lateral hot spot)

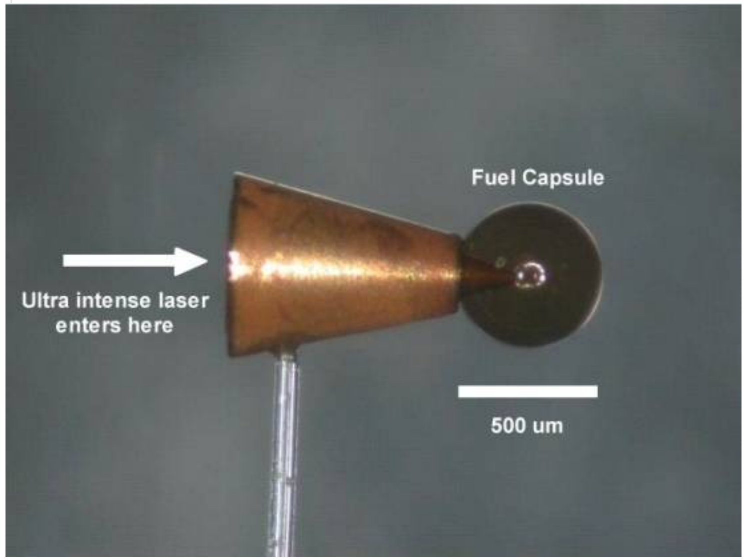


➢**Typical parameters: $E \approx 10$ kJ, $\Delta t \approx 10$ ps, $R \approx 10\mu m$, $E_{fast} \approx 1$ MeV**

Inertial Fusion- Fast Ignition

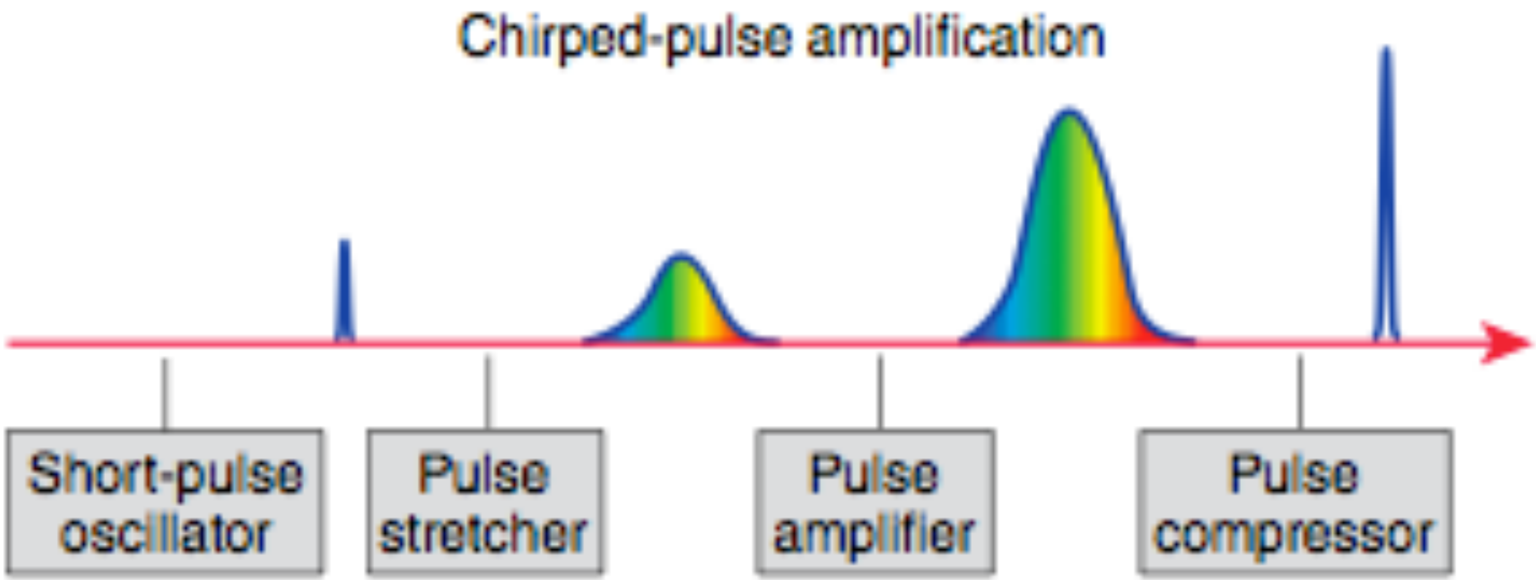






High-intensity lasers (high energy – short pulse)

Laser-plasma interaction in the “relativistic regime”



Evolution of laser performance:

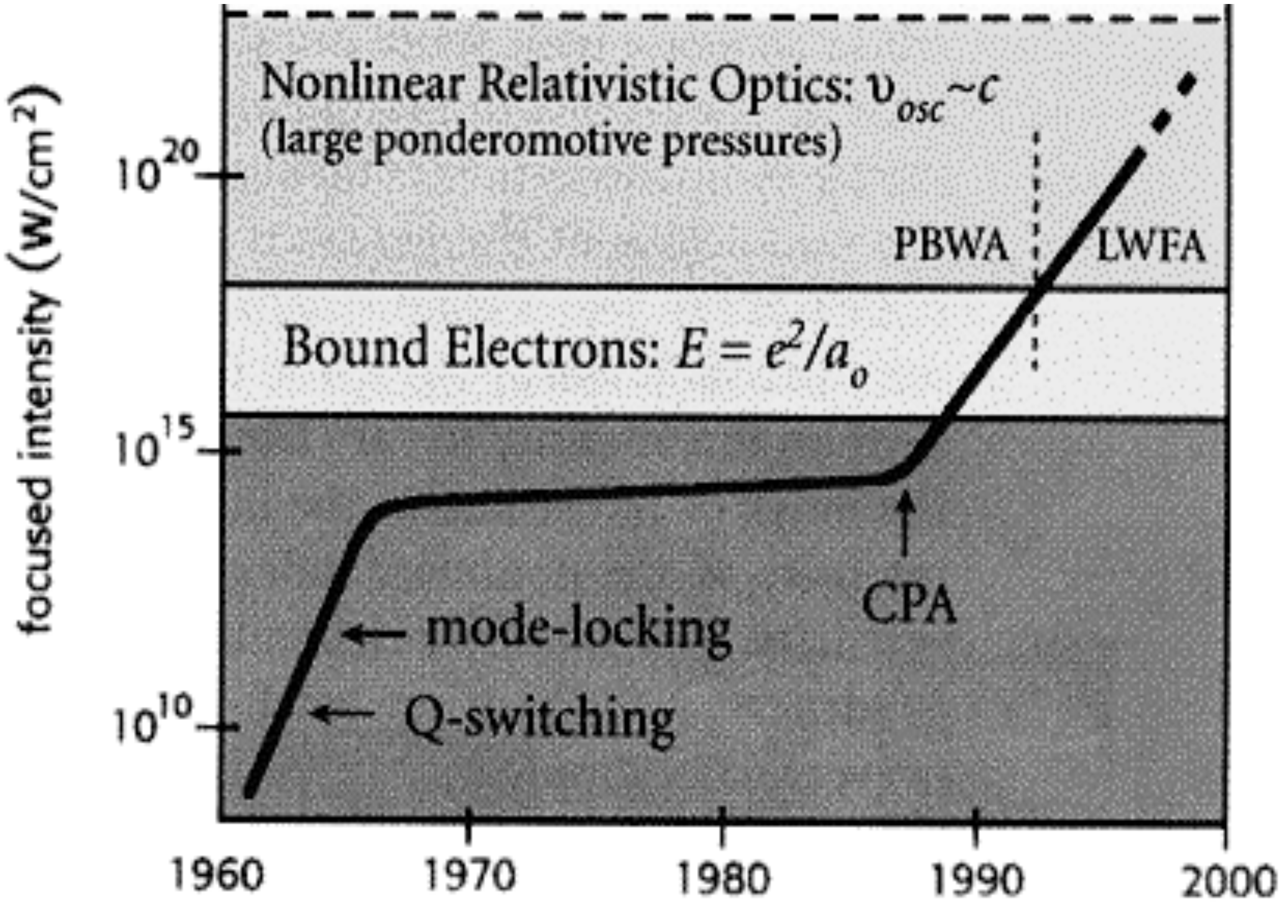
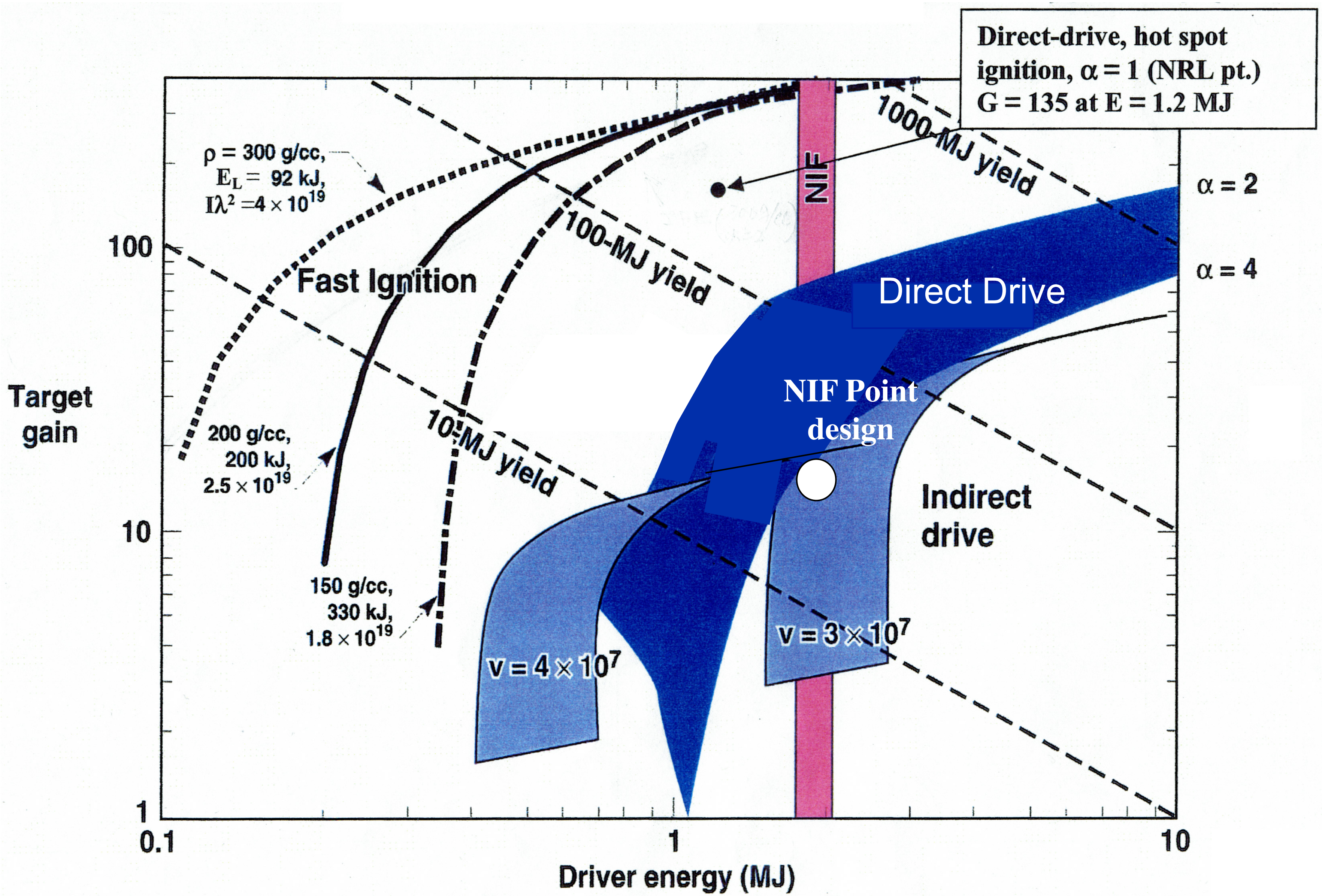


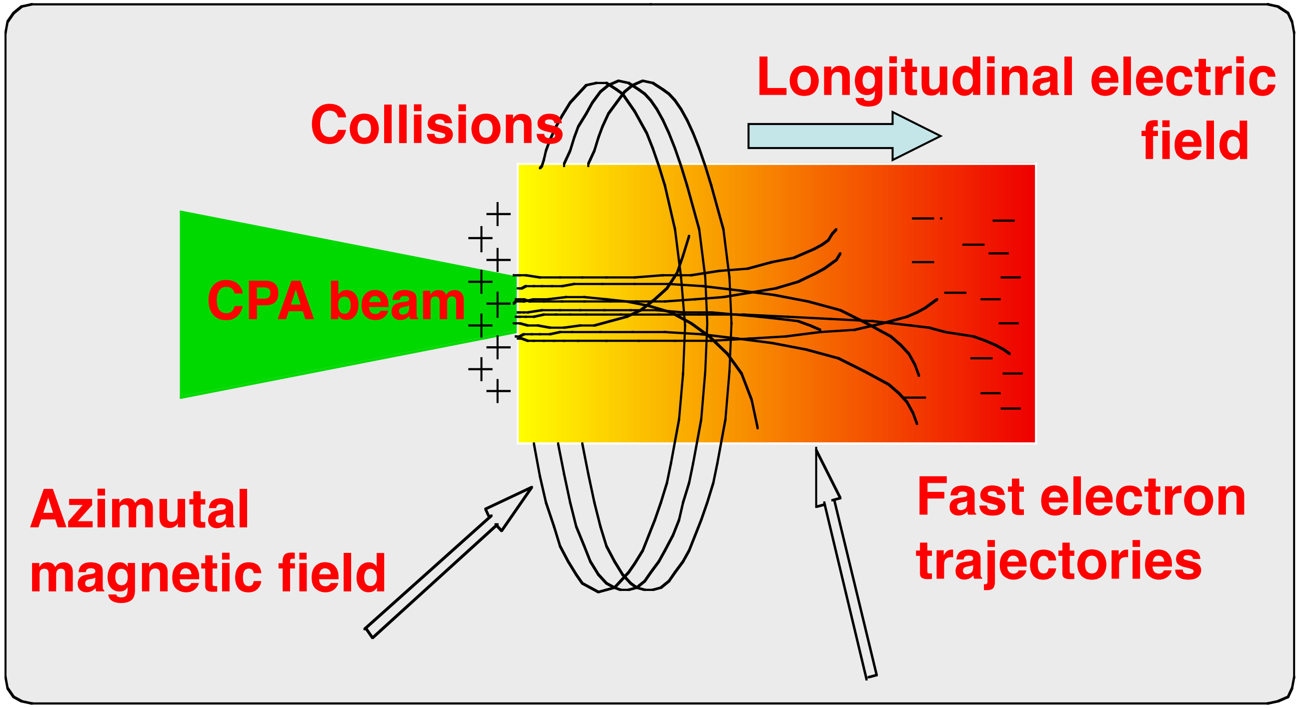
Figure 1. Laser intensity as a function of year, showing the impact of the CPA concept and the different thresholds of physical regimes. The sharp increase in intensity since the advent of CPA is comparable to the sharp increase after the invention of the laser in the 1960's.

Advantage of FI



Study of propagation:

Propagation of fast electrons in matter between n_c and 100 n_c over 200 - 300 μm is critical for the feasibility of fast ignition

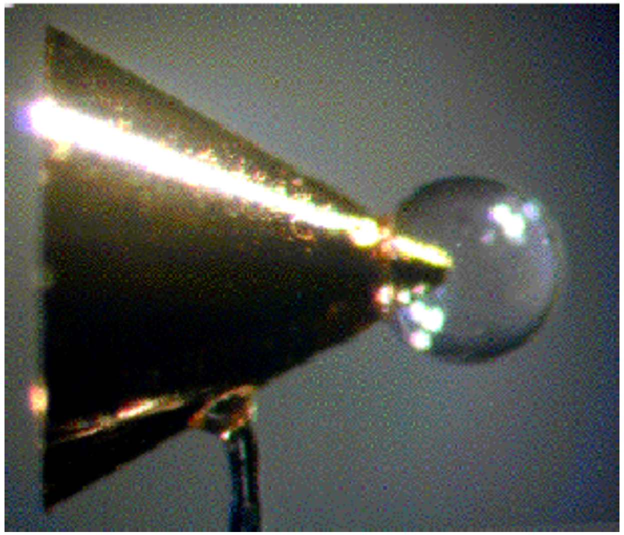


- **Collisional Effects (Stopping Power)**
- **Electric field effects**
- **Magnetic fields effects**

NEW PHYSICAL PROBLEM: VERY DIFFERENT FROM JUST BETHE-BLOCH

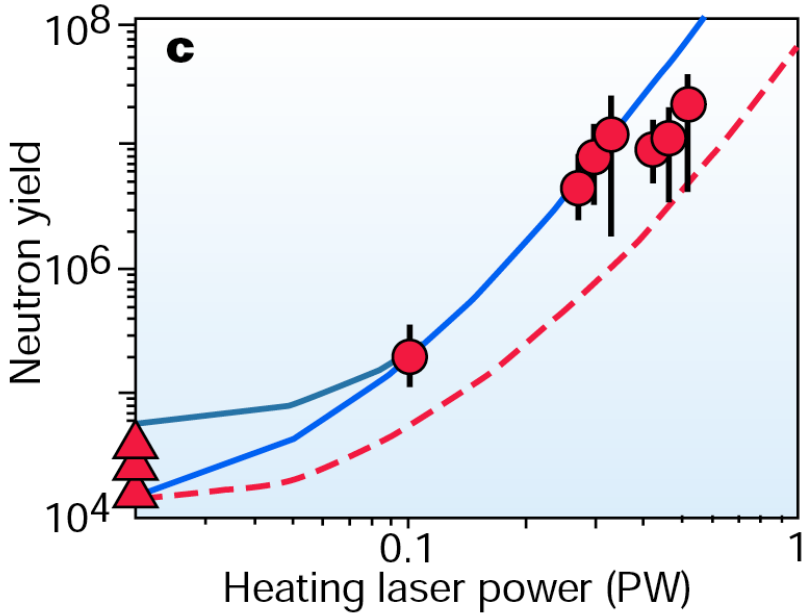
Use of cone targets

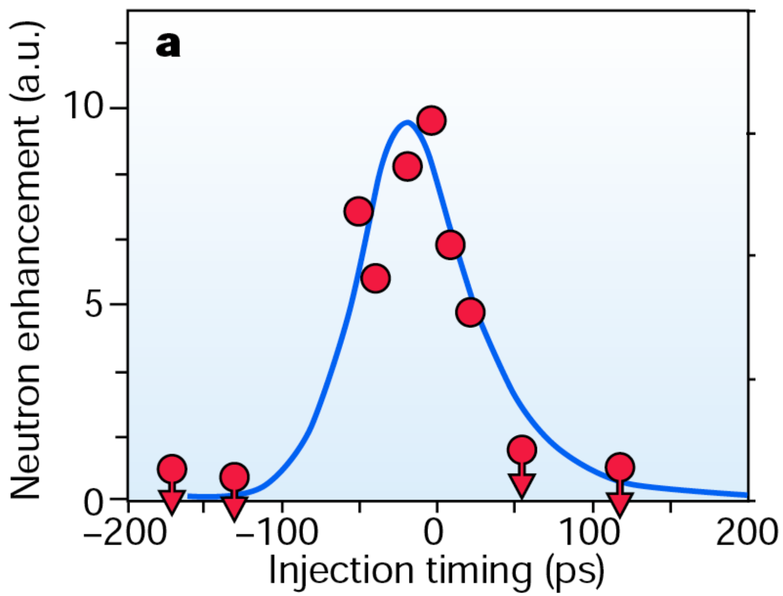
"Cone guided" targets were tested
at the ILE in Osaka



- The laser can interact in regions at higher densities
- Increment in neutron yield from compressed fusion targets

R. Kodama et al., Nature 418, 933 (2002)





Physical issues

- Cones do not seem viable for future reactors (same problems of Indirect Drive targets)
- The presence of the cone may prevent a sufficiently uniform compression and produce high-Z pollution
- How to control electron beam energy?
- Is there any tools to control electron beam divergence?

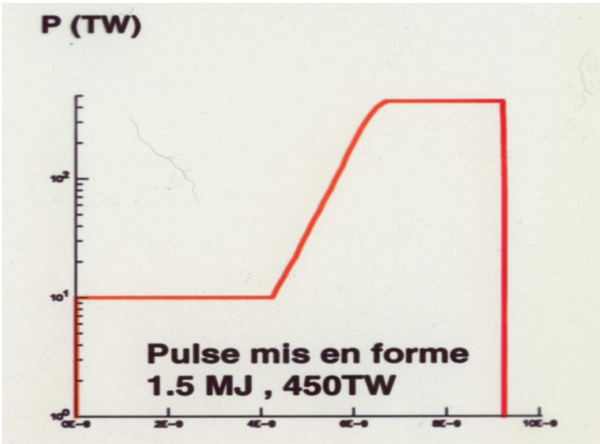
Although a very promising idea, FI is in a "premature" stage for what concerns technological developments (we need a 100 kJ - 10 ps laser beam)

[the main issue here is *scalability of collective effects*...]

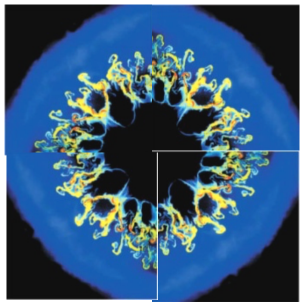
Shock ignition: a final laser spike launches a converging shock

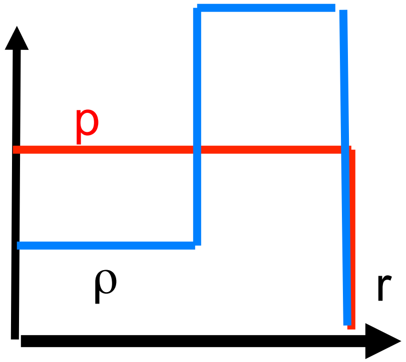
Conventionnal direct drive

450 TW, 1.5MJ pulse



High Aspect ratio target
V ~400 km/s

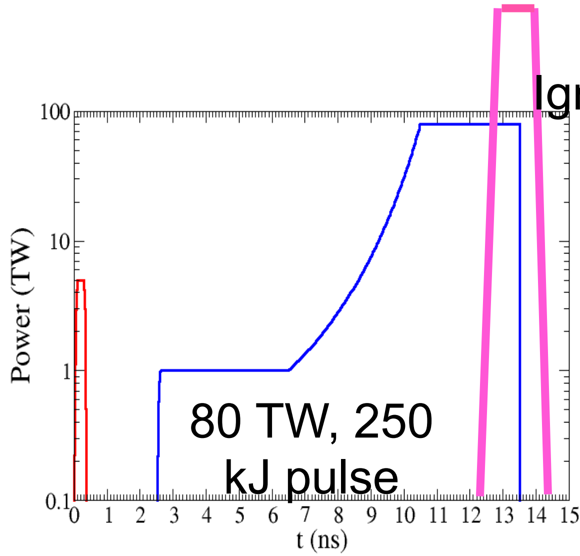




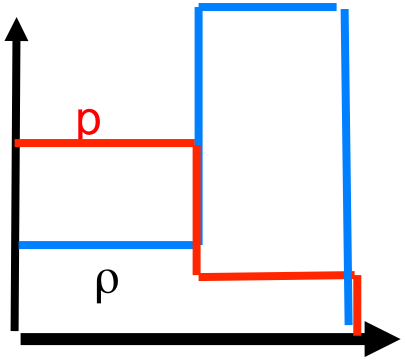
Produces an Isobaric fuel assembly

Low velocity drive

220 TW
Ignition spike



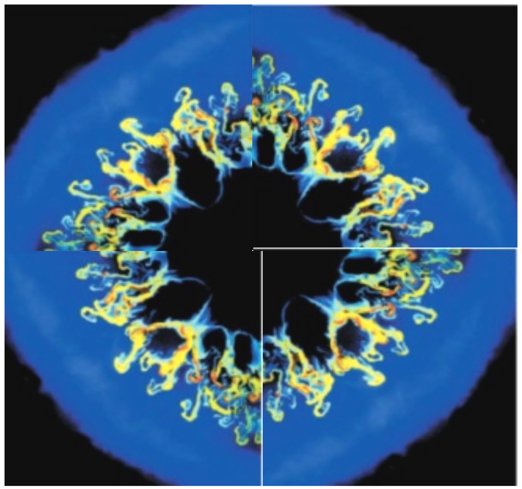
Low AR V~240 km/s



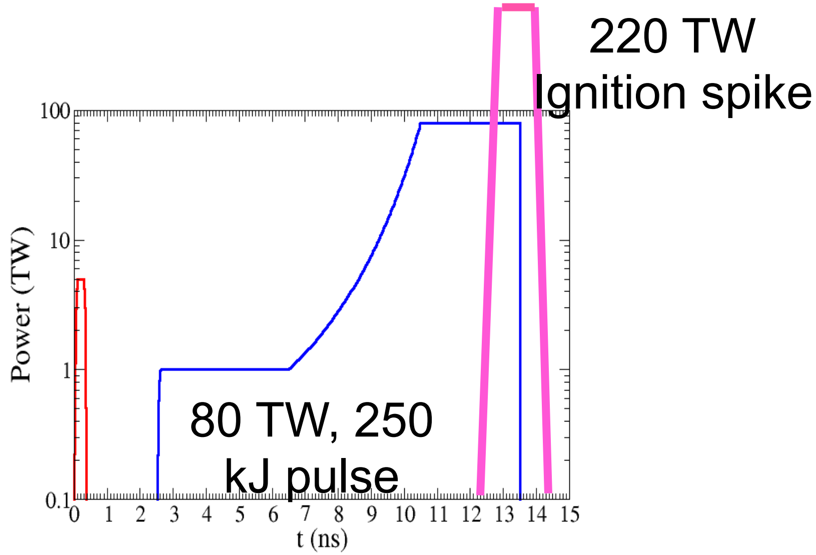
Fuel assembly is non isobaric

Shock ignition is less sensitive to hydro instabilities

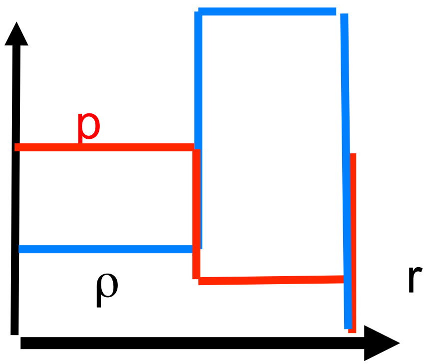
In SI, you do not create the hot spot with the "main" compression beam. Hence you do not need such high implosion velocity. Hence you can implode a more massive thicker shell which does not break due to RT



Low velocity drive



Low AR V~240 km/s



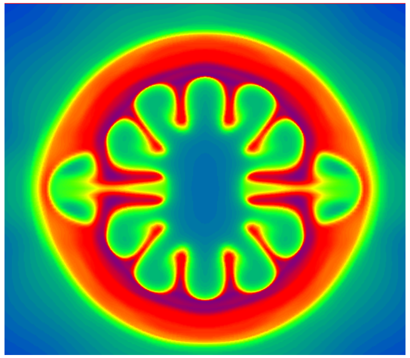
Fuel assembly is non isobaric

The Ignition shock mitigates RT growth at stagnation

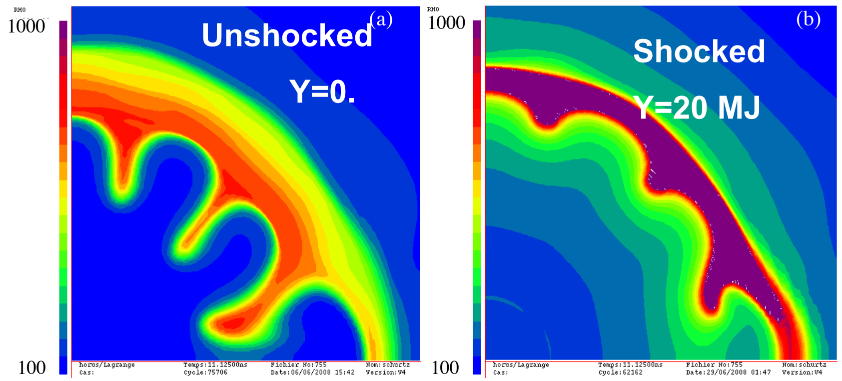
HiPER target at time of maximum ρR (1D)

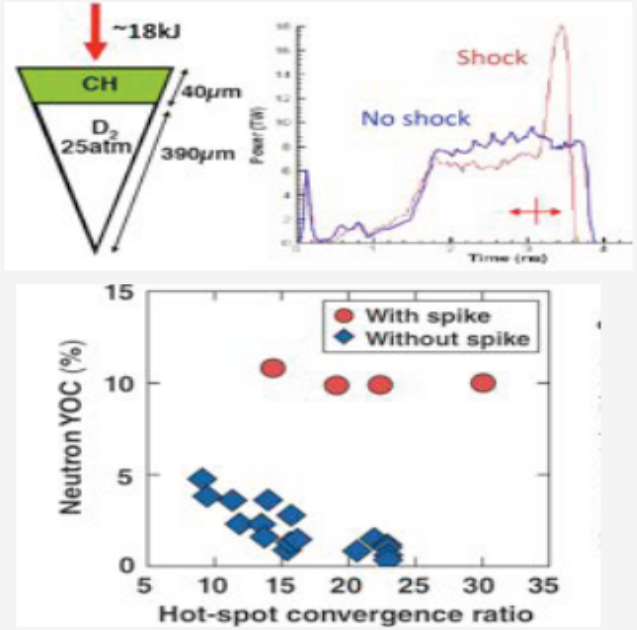
180 kJ

48 beams



HiPER target at ignition time





Is this experimental evidence of shock RT mitigation?

- OMEGA shock driven target
- D2 gas with CH shell
- Same total energy: shock, no shock

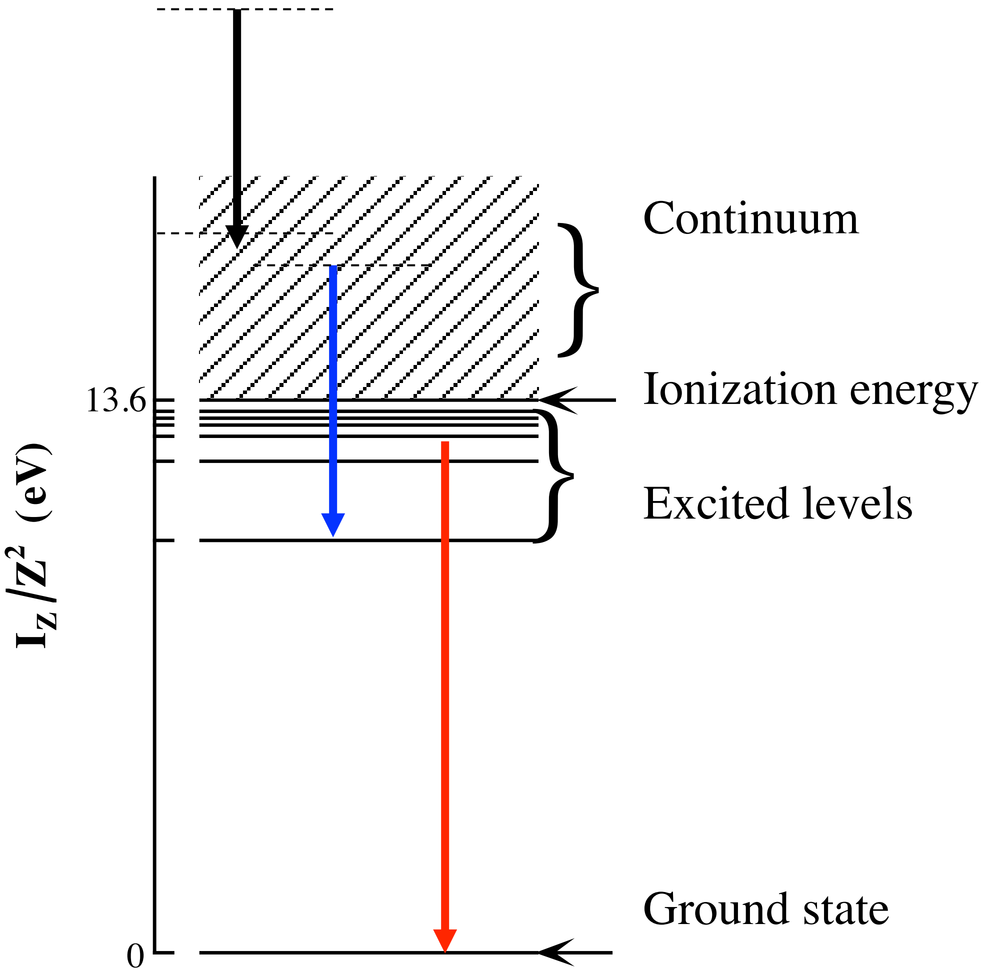
W. Theobald et al.

3 types of radiation emission in a plasma

Free-Free emission (continuum) or “bremsstrahlung”

Free-Bound emission
(continuum)
“recombination”

Bound bound emission
(lines)



Bremsstrahlung emission

For a Maxwellian population of free electrons the shape of bremsstrahulng emission is exponential. The quantity g_{ff} is the Gaunt factor (which has a quantum origin)

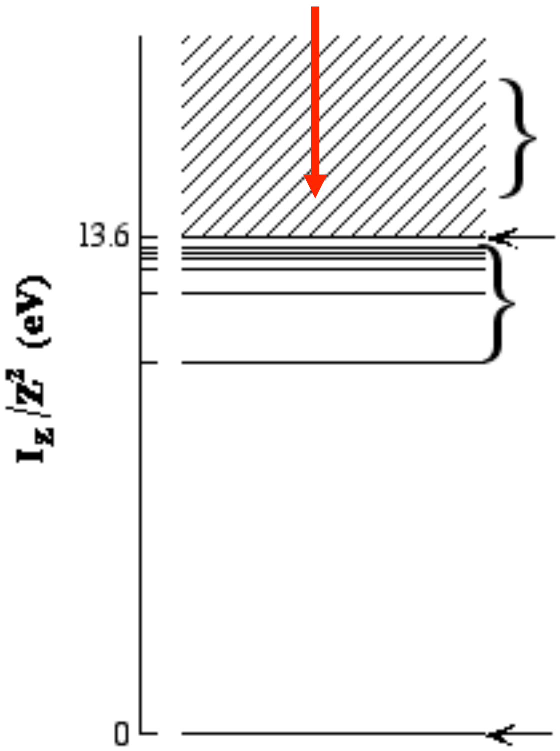
$$\frac{d\varepsilon_{ff}}{d(h\nu)} = 1.5\times10^{-32}\frac{n_e}{T_e^{1/2}} g_{ff} \exp(-h\nu/T_e)\sum_i N_i Z_i^2 = J_o(T_e)\exp(-h\nu/T_e)$$

The total power emitted (W/cm^3) is

$$P_{ff} = 1.5\times10^{-32}\frac{n_e}{T_e^{1/2}}\sum_i N_i Z_i^2$$

When it is expressed in wavelength there is a maximum at

$$\lambda(\max) \approx \frac{6200}{\mathrm{T_e}}$$



Bremsstrahlung emission

Attention: the important parameter in plasma emission is not the average ionization degree but the effective ionization degree (higher charges contribute more to emission)

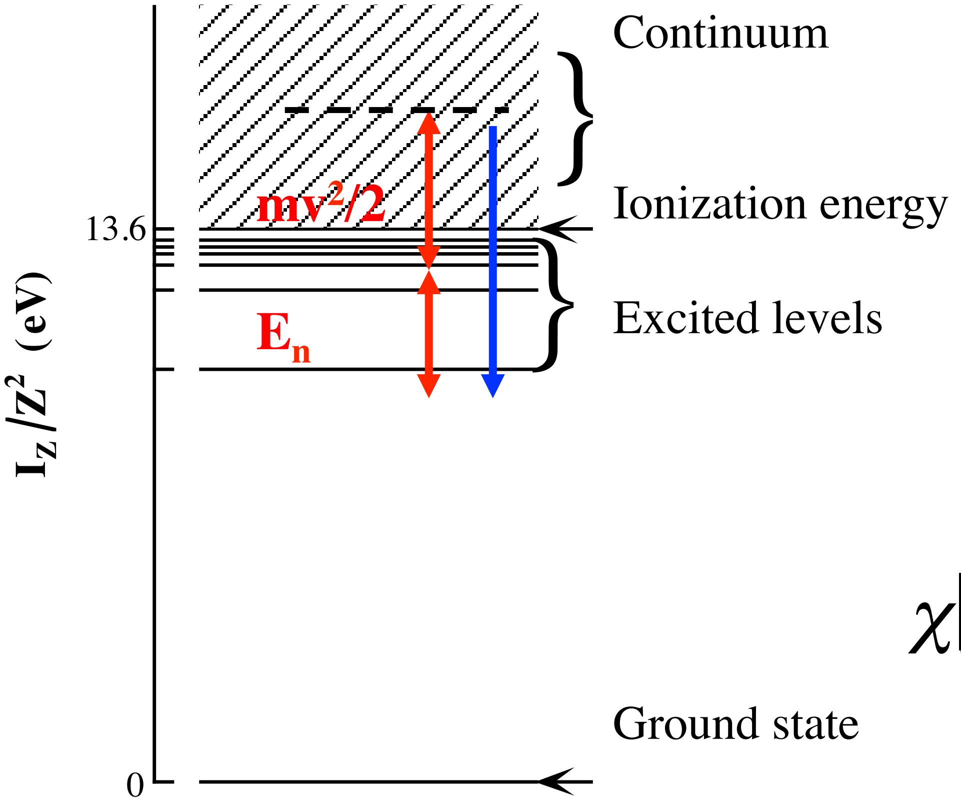
$$\frac{d\varepsilon_{ff}}{d(h\nu)} = 1.5 \times 10^{-32} \frac{n_e}{T_e^{1/2}} g_{ff} \exp(-h\nu/T_e) \sum_i N_i Z_i^2 = J_o(T_e) \exp(-h\nu/T_e)$$

$$Z^* = \frac{1}{N_{tot}} \sum N(Z) Z$$

$$Z_{eff} = \frac{1}{N_{tot}} \sum N(Z) Z^2$$

Recombination emission

$$\frac{d\varepsilon_{\mathrm{fb}}}{d(h\nu)} = 1.5\times10^{-32}\,\frac{n_e}{T_e^{3/2}}\sum_i N_i Z_i^2 \sum_{n=1}^{\infty}\frac{J_i^{(n)}}{n^3}\,g_{fb}^{(n)}\,I_i^{(n)}\exp\left[-\frac{\left(h\nu - I_i^{(n)}\right)}{T_e}\right]\chi\left(h\nu - I_i^{(n)}\right)$$

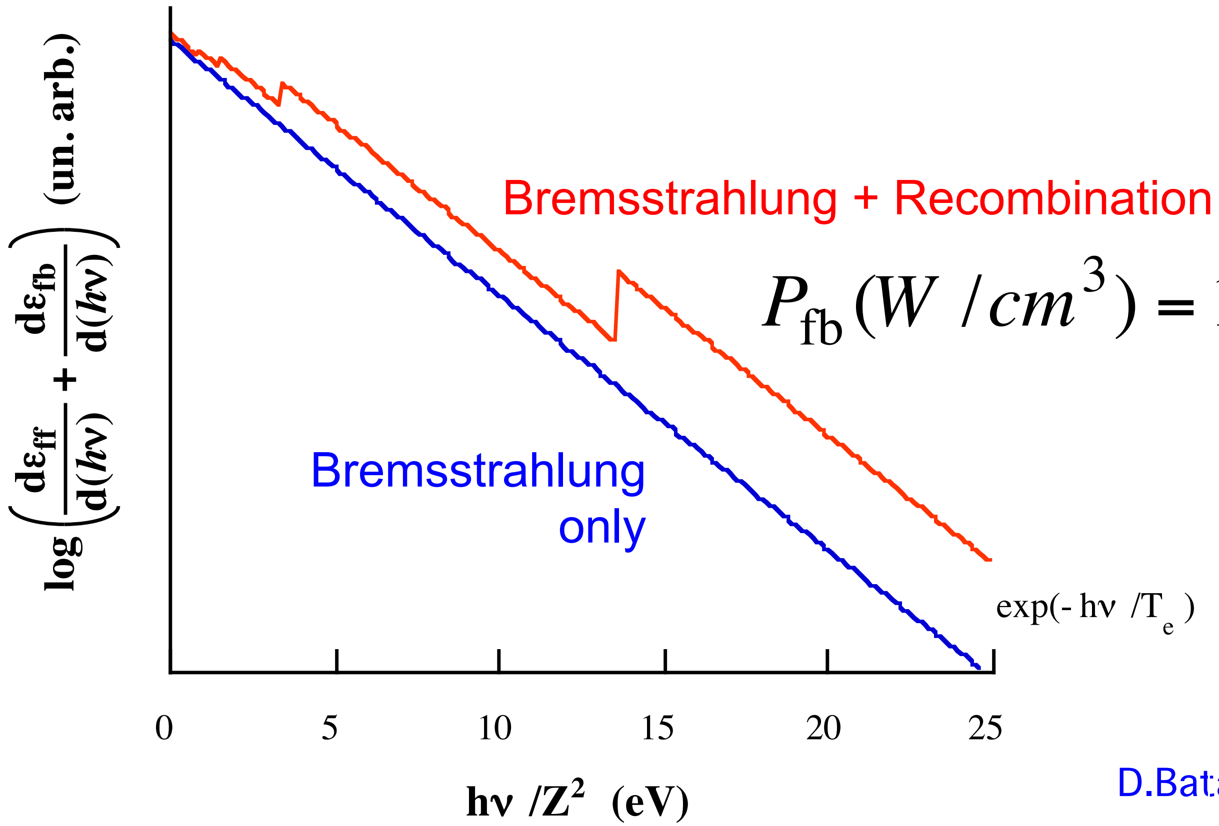


$$h\nu = \frac{1}{2}m_e v_e^2 + E_n^{Z_i}$$

$$\chi\left(h\nu - I_i^{(n)}\right) = \begin{cases} 0 \text{ per } h\nu < I_i^{(n)} \\ 1 \text{ per } h\nu \geq I_i^{(n)} \end{cases}$$

Recombination emission

A photon is emitted when an ion with charge Z_i captures an electron. Initial states have a continuous distribution implying a continuum spectrum. However spectra are characterized by jumps corresponding to recombination edges



$$P_{\mathrm{fb}}(W/cm^3) = 1.5 \times 10^{-32} \frac{n_e}{T_e^{3/2}(eV)} \times \sum_i N_i Z_i^2 \sum_{n=1}^{\infty} I_i^{(n)}$$

Minimal temperature for ignition

In a hydrogen plasma (DT) at high temperature the only emission is due to bremsstrahlung. If n_e is in cm^{-3} then

$$W_{\mathrm{B}} \simeq C_B n_i n_e T_h^{1/2} = 5.35 \times 10^{-31} n_e^2 T_{h\mathrm{keV}}^{1/2}\ \mathrm{W/cm^3}$$

The power per unit volume produced by nuclear fusion reaction which remains inside the plasma is

$$W_\alpha = \frac{1}{5}\varepsilon_{DT} n_D n_T \langle \sigma_{DT} v \rangle .$$

These are BOTH powers per unit volume!
For ignition clearly we must have

$$W_\alpha > W_B$$

Minimal temperature for ignition

Using

$$\langle \sigma_{DT} v \rangle \simeq C_{DT} T^2 = 1.1 \times 10^{-18} T_{\mathrm{keV}}^2 \ \mathrm{cm}^3/\mathrm{s}.$$

We get

$$W_\alpha = \frac{1}{20} \varepsilon_{DT} n_i^2 C_{DT} T^2$$

Since $P = n_i T$, we see that W_α depends on plasma pressure only.
By considering $W_\alpha > W_B$ we see that for temperature

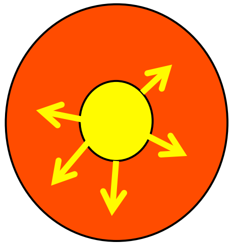
$$\frac{1}{20} \varepsilon_{DT} C_{DT} T^2 > C_B T^{1/2}$$

$$T > T_{Post} = \left(\frac{20\ C_B}{\varepsilon_{DT} C_{DT}} \right)^{2/3} \approx 5\ keV$$

Electron losses

The central hot spot loses energy by electron thermal conduction to the colder surrounding shell.
The (electronic) thermal flux (power transported per unit area) is related to temperature gradient by Fourier's law (here T_h refers to the temperature of hot spot)

$$\vec{q}_e = k\nabla T_h$$



In microscopic terms

$$k = n_e \mathrm{v}_{th} \lambda c_{\mathrm{v}} / 3N_A = n_e \mathrm{v}_{th} \lambda / 2 \qquad \left(c_{\mathrm{v}} = \frac{3}{2} N_A \right)$$

and the mean free path λ is related to the thermal velocity and the collision frequency

$$\lambda = \mathrm{v}_{th} / \nu_{ei}$$

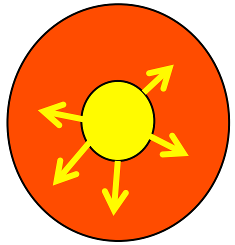
$$\mathrm{v}_{th} = \left(T_e / m_e \right)^{1/2} = 4.19 \times 10^7 T_e^{1/2} \qquad cm/s$$

Electron losses

Hence

$$k = n_e \mathrm{v}_{th} \lambda / 2 = n_e \mathrm{v}_{th}^2 / 2\nu_{ei} = n_e T_e / 2 m_e \nu_{ei}$$

And using Spitzer' theory



$$\nu_{ei} = 2.91 \times 10^{-6} \ln \Lambda T_e^{-3/2} (eV) \qquad \sec^{-1}$$

$$k = k_o T_e^{5/2} = 3 \times 10^{12} \; T_e^{5/2} \qquad \left(\mathrm{W\ cm^{-1} keV^{-7/2}} \right)$$

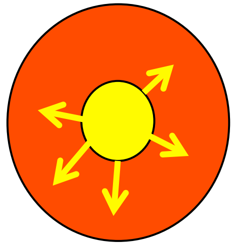
We can now write the energy balance as

$$\int (W_\alpha - W_B) 4\pi r^2 \, dr = \int q_e \, dS = \int k_o T_e^{5/2} \nabla T_e dS$$

Electron losses

Applying the theorem of divergence in spherical coordinates

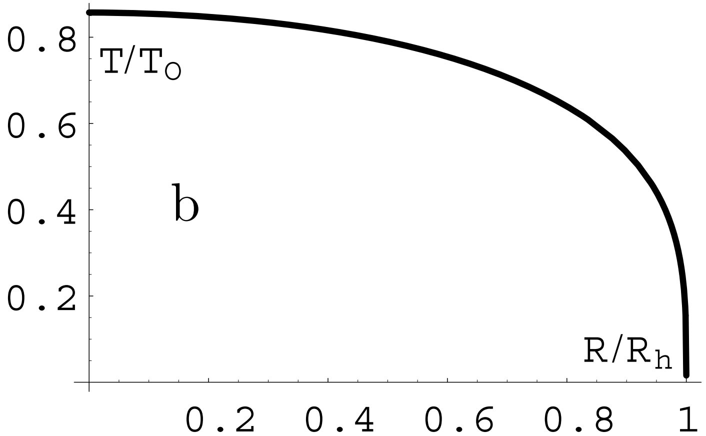
$$\int q_e dS = \int 4\pi r^2\, dr \left(\nabla \cdot q_e\right)$$



We obtain a differential equation

$$W_\alpha - W_B = \nabla \cdot q_e = -\frac{k_o}{r^2}\frac{d}{dr}\left(r^2 T_h^{5/2}\frac{dT_h}{dr}\right) \qquad (**)$$

Which can be explicitly solved in the limit of high temperatures ($T >> T_{post}$) and neglecting radiative losses ($W_B \approx 0$) allowing to find the temperature profile in the hot spot and bringing to the result:

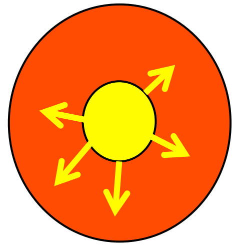


$$q_e(R_h) = 0.57 k_o T_h^{7/2} / R_h$$

Electron losses

Using a simplified approach

$$\left(W_\alpha - W_B\right)\frac{4}{3}\pi R_h^3 = 4\pi R_h^2 q_e(R_h)$$



Looking dimensionally at the rigth side of eq. (**) we see that $q_e(R_h) \approx k_o T^{7/2}/R^h$. For better precision we take the exact solution of the differential equation with the numerical factor 0.57

$$\left(W_\alpha - W_B\right)\frac{R_h}{3} = 0.57 k_o T_h^{7/2} / R_h$$

$$W_\alpha = \frac{1}{20}\varepsilon_{DT} n_i^2 C_{DT} T^2$$

$$W_B = C_B n_i n_e T^{1/2} = C_B Z^* n_i^2 T^{1/2}$$

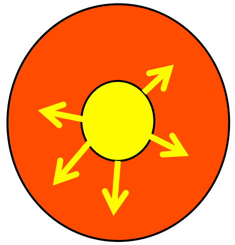
Electron losses

$$\frac{1}{20}\varepsilon_{DT} n_i^2 C_{DT} T_h^2 - C_B Z^* n_i^2 T_h^{1/2} = 1.71 k_o T_h^{7/2} / R_h^2$$

$$n_i = \rho_h / m_i$$

$$(\rho_h R_h)^2 = \frac{1.71 k_o T_h^{7/2} m_i^2}{\varepsilon_{DT} C_{DT} T^2 / 20 - C_B Z^* T^{1/2}} =$$

$$\left(\frac{34 k_o m_i^2}{\varepsilon_{DT} C_{DT}}\right) \frac{T_h^{7/2}}{T_h^2 - 20 C_B Z^* T_h^{1/2} / \varepsilon_{DT} C_{DT}}$$



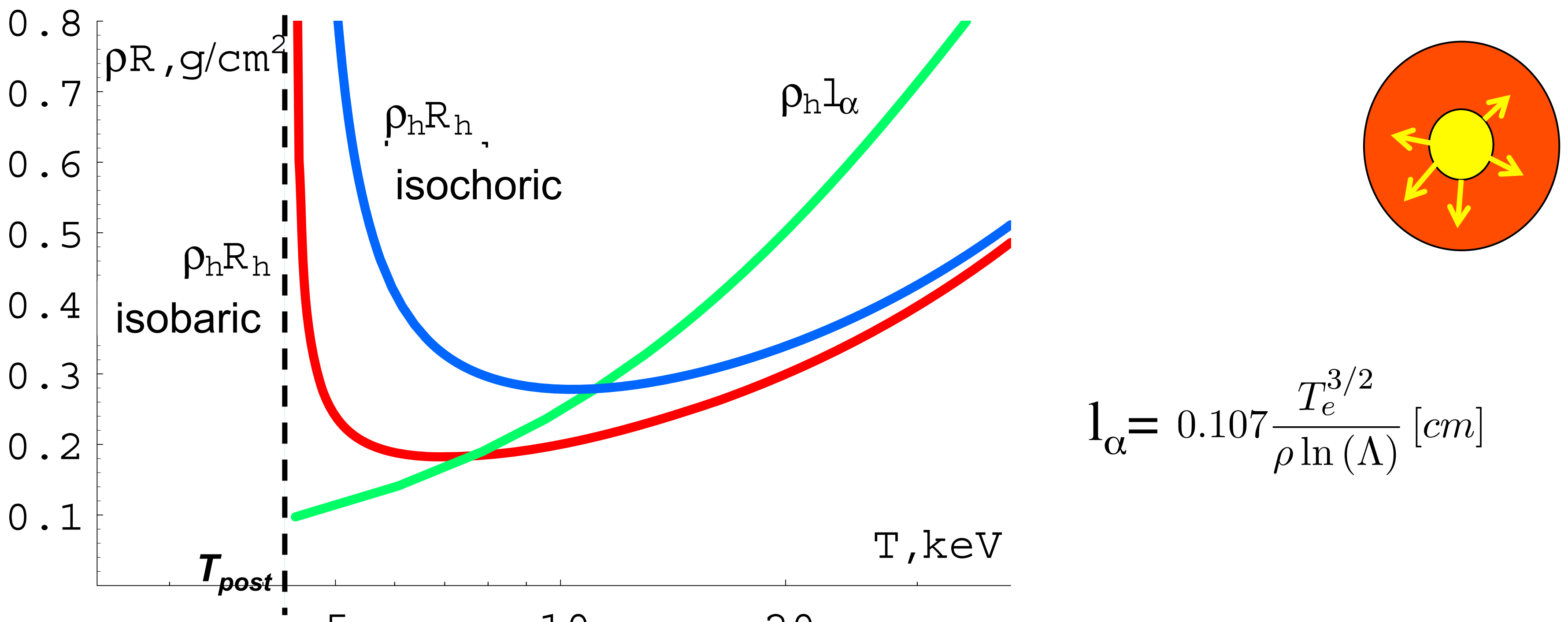
$$T_{Post} = \left(\frac{20\ C_B}{\varepsilon_{DT} C_{DT}}\right)^{2/3} \qquad Z^* = 1$$

$$(\rho_h R_h)^2 = \left(\frac{34 k_o m_i^2}{\varepsilon_{DT} C_{DT}}\right) \frac{T_h^{7/2}}{T_h^2 - T_h^{1/2} T_{Post}^{3/2}} = \left(\frac{34 k_o m_i^2}{\varepsilon_{DT} C_{DT}}\right) \frac{T_h^3}{T_h^{3/2} - T_{Post}^{3/2}}$$

$$\rho_h R_h = \left(\frac{34 k_o m_i^2}{\varepsilon_{DT} C_{DT}}\right)^{1/2} \frac{T_h^{3/2}}{\sqrt{T_h^{3/2} - T_{Post}^{3/2}}}$$

Electron losses

Drawing this function: $$\rho_h R_h = \left(\frac{34 k_o m_i^2}{\varepsilon_{DT} C_{DT}}\right)^{1/2} \frac{T_h^{3/2}}{\sqrt{T_h^{3/2} - T_{Post}^{3/2}}}$$

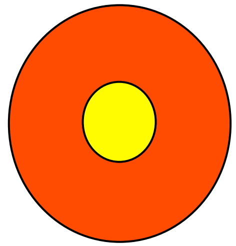


$$l_\alpha = 0.107 \frac{T_e^{3/2}}{\rho \ln(\Lambda)} \, [cm]$$

$$(\rho_h R_h)_{\min} \approx 0.2 g/cm^2 \qquad T_h \approx 7 keV$$

➡ **Lawson's criterium for the hot spot**

Optimal conditions for the hot spot



Since

$$\left(\rho_h R_h\right)_{\min} \approx 0.2 g / cm^2 \qquad T_h \approx 7 keV$$

Then

$$\varepsilon_{chauf} = 110 M_h T_h = 110\left(\frac{4}{3}\pi R_h^3 \rho_h\right) T_h = 110 \frac{\left(R_h \rho_h\right)^3}{\rho_h^2}\left(\frac{4}{3}\pi T_h\right) =$$

$$= 42 \times \frac{(0.2 g / cm^2)^3}{\rho_h^2} \times (7 keV) = \frac{22}{\rho_h^2} MJ$$

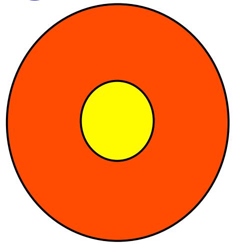
The isobaric condition allows to know the pressure and the density in the compressed shell material

$$p_h = \rho_h T_h / m_i = A_h \rho_h T_h \qquad A_h = 770 (Mbar / keV / g / cm^3)$$

$$p_F = \alpha A_F \rho_F^{5/3} = 2.16 \ \alpha \ \rho_F^{5/3} \quad MBar$$

$$p_h = p_F \qquad \Rightarrow \rho_F = (A_h \rho_h T_h / \alpha A_F)^{3/5} \approx 110 (\rho_h / \alpha)^{3/5}$$

Energy spent in compression and heating



$$(\rho_h R_h)_{\min} \approx 0.2 g/cm^2 \qquad T_h \approx 7keV$$

$$\rho_F = (A_h \rho_h T_h / \alpha A_F)^{3/5} \approx 110(\rho_h/\alpha)^{3/5}$$

$$\varepsilon_{chauf} = 110 M_h T_h = \frac{22}{\rho_h^2} MJ$$

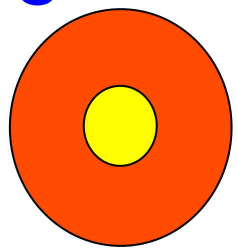
$$\varepsilon_{compr} = 0.35 \alpha M_f \rho_F^{2/3} \quad MJ$$

$$\varepsilon_{tot} = \varepsilon_{chauf} + \varepsilon_{compr} = \frac{22}{\rho_h^2} + 0.35\alpha M_f \left(110(\rho_h/\alpha)^{3/5}\right)^{2/3} =$$

$$= \frac{22}{\rho_h^2} + 7.2\alpha^{3/5} M_f \rho_h^{2/5} \quad MJ$$

Energy spent in compression and heating

$$\varepsilon_{tot} = \varepsilon_{chauf} + \varepsilon_{compr} = \frac{22}{\rho_h^2} + 7.2\alpha^{3/5} M_f \rho_h^{2/5} \quad MJ$$

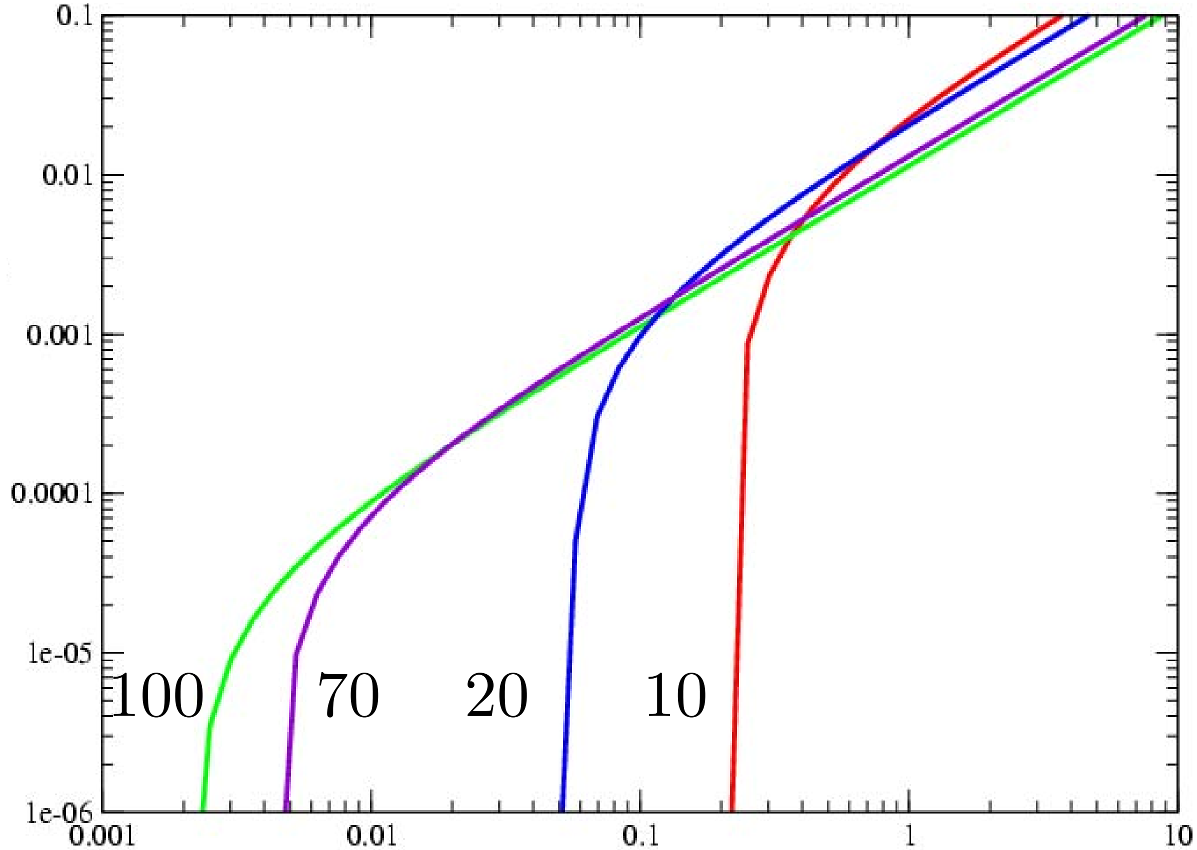


$$M_f = \frac{\varepsilon_{tot} - 22/\rho_h^2}{7.2\alpha^{3/5}\rho_h^{2/5}}$$

$$\varepsilon_{threshold} = 22/\rho_h^2$$

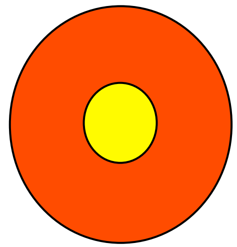
$$\varepsilon_{tot} >> \varepsilon_{threshold} \Rightarrow$$

$$M_f = \frac{\varepsilon_{tot}}{7.2\alpha^{3/5}\rho_h^{2/5}}$$



Gain from fusion

$$\left(\rho_h R_h\right)_{\min} \approx 0.2 g/cm^2 \qquad T_h \approx 7keV \qquad \alpha = 3$$



$$G_{fus} = \varepsilon_{fus} / \varepsilon_{tot}$$

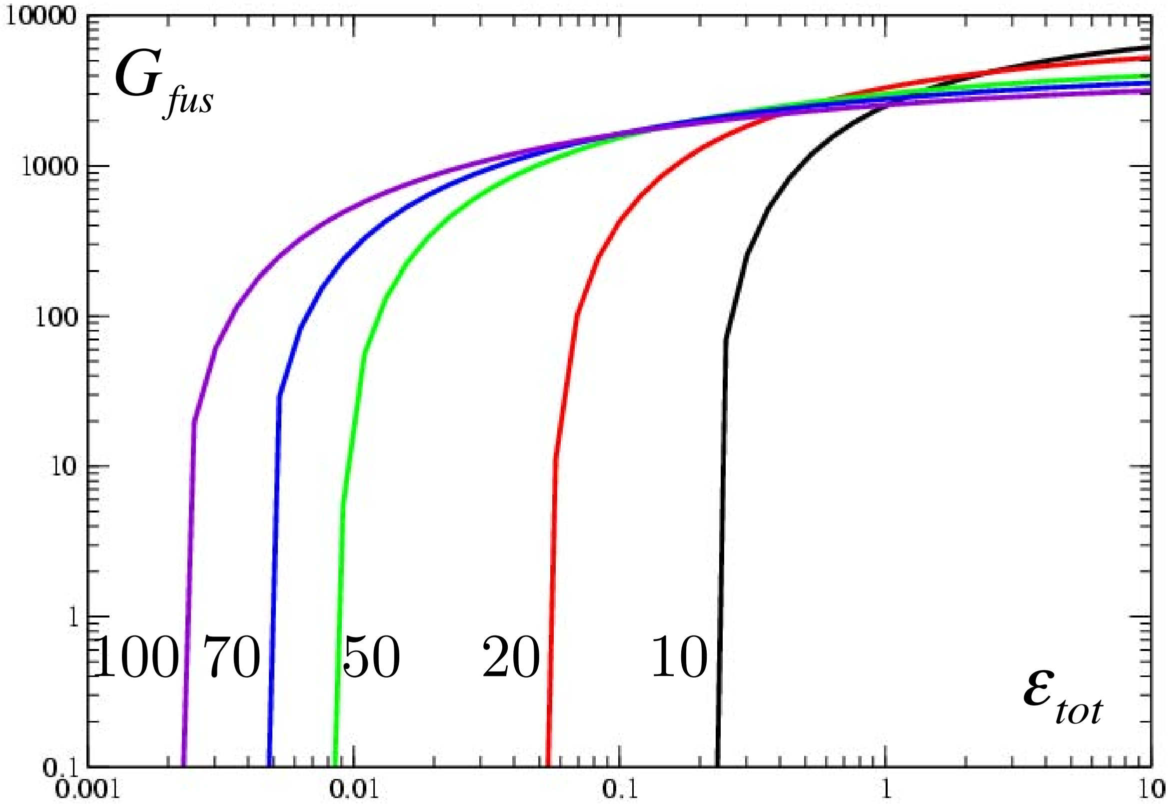
$$\varepsilon_{tot} >> \varepsilon_{threshold} \Rightarrow$$

$$\varepsilon_{tot} = 7.2\alpha^{3/5}\rho_h^{2/5}M_f$$

$$\varepsilon_{fus} = \varepsilon_{DT}N_f = \frac{M_f}{2m_i}\Phi_B\varepsilon_{DT}$$

$$= 3.4\times10^5\Phi_B M_f$$

$$G = 4.7\times10^4\Phi_B\alpha^{-3/5}\rho_h^{-2/5}$$



CELIA

Gain from fusion

$$\rho_h R_h \approx 0.2 g/cm^2 \qquad T_h \approx 7keV \qquad \alpha = 3$$

$$G_{fus} = \varepsilon_{fus} / \varepsilon_{tot}$$

$$\varepsilon_{tot} >> \varepsilon_{threshold} \Rightarrow \quad \varepsilon_{tot} = 7.2\alpha^{3/5}\rho_h^{2/5}M_f$$

$$\Rightarrow \rho_h^{2/5} = \varepsilon_{tot} / 7.2 M_f \alpha^{3/5}$$

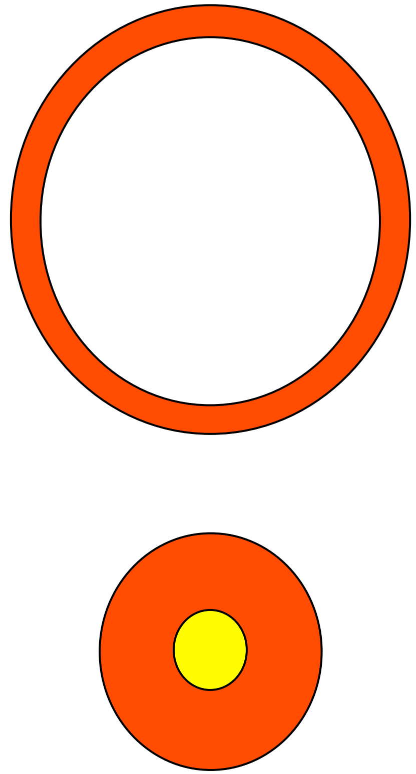
$$\varepsilon_{fus} = 3.4 \times 10^5 \Phi_B M_f$$

$$G = 4.7 \times 10^4 \Phi_B \alpha^{-3/5} \rho_h^{-2/5} = 3.4 \times 10^5 \Phi_B M_f / \varepsilon_{tot}$$

$$\varepsilon_{tot} = 0.1\, MJ \qquad M_f = 1\ mg \qquad \Phi_B = 0.33$$

$$G = 1100 \qquad \Rightarrow \qquad \varepsilon_{fus} = 110 MJ$$

$$if \quad \varepsilon_{tot} = 0.1 \varepsilon_{laser} \quad then \qquad G_{tot} \approx 110$$



Following topics

1) How the target is really imploded. The “rocket model” in ICF
2) A simple self-similar model of laser-produced plasmas (laser ablation and shock pressure)
3) Laser-produced shocks and their dynamics
4) Experiments on warm dense matter and High Energy Density
5) Diagnostics for inertial fusion and laser-produced plasmas

Correction of first test / 1st question

1) calculate the minimum approach distance between D and T ions in a plasma with T ≈ 1 keV. Assume velocity equal thermal velocity for both species.

What is the minimum approach distance for D – D ions?

Assume a maxwellian distribution. What is the minimum approach distance for D and T ions with velocity equal to 3 times thermal velocity?

Correction of first test / 1st question

$$E_{kin} = \frac{mV^2}{2} = \frac{3T}{2}$$

$$E_{coul} = \frac{e^2}{r}$$

$$E_D + E_T = 3T = \frac{e^2}{r}$$

$$r = \frac{e^2}{3T}$$

$$T = 1keV = 1.6\times 10^{-9} erg$$

$$e = 4.8\times 10^{-10} cgs$$

$$r = 4.8\times 10^{-11} cm$$

For comparison

$$r_H = 0.529\ A \approx 5\times 10^{-9} m$$

$$r_p = 0{,}84184\ \text{fm} \approx 10^{-13} cm$$

$$r_i = 1.4\times 10^{-15} A^{1/3}\ \text{m}$$

Notice that V is different for D and T ions but kinetic energy and minimal distances are the same for both (bigger mass implies smaller velocity but bigger inertia)

If V is 3 times bigger, E_{kin} is 9 times bigger and r is 9 times smaller

Correction of first test / 2nd question

2) using the principle of conservation of energy and momentum show that in a DT reaction the energy is shared as 1/5 and 4/5 between the α particle and the neutron

$$E = 17.6 MeV = E_\alpha + E_n$$

$$0 = p_\alpha + p_n$$

Notice that this is true in the centre of mass frame.
In the actual laboratory frame, energies can be shifted due to the initial total energy

$$m_\alpha = 4 m_n$$

$$v_\alpha = \frac{m_n}{m_\alpha} v_n$$

$$E_\alpha = \frac{m_\alpha v_\alpha^2}{2} = \frac{4 m_n}{2}\left(\frac{v_n}{4}\right)^2 = \frac{1}{4} E_n$$

Correction of first test / 3rd question

3) calculate the disassembly time of a sphere of compressed DT at the temperature of 10 keV and radius 100 μm.
Does such time depend on the density of the compressed fuel?

$$c_s = \sqrt{\frac{\gamma Z T_e}{m_i}} = 9.79 \times 10^5 \sqrt{\frac{\gamma Z T_e(eV)}{A}} cm/s \approx 7 \times 10^7 cm/s$$

$$\tau = \frac{R}{4c_s} = \frac{100\,\mu m}{28 \times 10^7 cm/s} = \frac{10^{-2}}{28 \times 10^7} = 1.4 \times 10^{-10} s = 36 ps$$

Notice that this is independent on the density, i.e. on achieved compression

Correction of first test / 4th question

4) calculate the mass of DT (50% -50%) satisfying Lawson's criterum at solid state density ($\rho = 0.25$ g/cm^3)
calculate the energy released if all the combustible is burned. Express such energy in equivalent ton of TNT

$$\rho r = 0.3 g/cm^2 \qquad \Rightarrow \qquad r = \frac{0.3}{0.25} = 1.2 cm$$

$$M = \frac{4}{3}\pi r^3 \rho = 1.8 g$$

$$\varepsilon_{fus} = 3.5 \times 10^5 \Phi_B M_f \;\; MJ = 6.3 \times 10^5 \;\; MJ$$

$$1 kton = 4.18 \times 10^{12} \; J = 4.18 \times 10^6 \;\; MJ$$

$$\varepsilon_{fus} = 0.15 \text{ kton}$$

$$if \quad M_f = 1 mg \quad \Rightarrow \quad \varepsilon_{fus} = 0.15 \; kgTNT$$

Correction of first test / 5th question

4) Let's take 10 mg DT at solid (cryogenic) density. What is the radius of the sphere? To which density must it be compressed on order to satisfy Lawson's criterium?

$$V = \frac{M}{\rho} = \frac{10^{-2}}{0.25} = 0.04 cm^3$$

$$r = \left(\frac{3V}{4\pi}\right)^{1/3} = 0.21 cm \quad \Rightarrow \rho r = 0.05 g/cm^2$$

$$\rho_{com} = \frac{M}{V_{com}} = \frac{3M}{4\pi r_{com}^3}$$

$$\rho_{com} r_{com} = \frac{3M}{4\pi r_{com}^2} = 0.3 \Rightarrow r_{com} = \sqrt{\frac{3M}{4\pi 0.3}} = 0.09 = 890 \mu m$$

$$V_{com} = \frac{4\pi r_{com}^3}{3} = 2.9\ 10^{-3} cm^3$$

$$\rho_{com} = \frac{0.01}{V_{com}} = 3.5 g/cm^3 = 13 \rho_{cryo}$$

Correction of first test / 6th question

6) consider the target of viewgraph 30. Assume that the initial density of DT gas corresponds to 1 atmosphere (standard conditions, T=0.025 eV). How many DT ions are contained in the gas?

INITIAL CONDITIONS

CRYOGENIC SHELL

- $R_{in} \approx$ 2 mm, $\Delta r \approx$ 33 μm
- $A = R_{in} / \Delta r \approx 60$
- $V_{in} \approx 4 \pi R_{in}^2 \Delta r \approx 1.6 \ 10^{-3}$ cm^3
- $\rho_{in} \approx 2.5 \times 0.1$ g/ cm^3, M $\approx$ 0.41 mg

$$V = \frac{4\pi r^3}{3} = 0.03 cm^3$$

$$V_{mol} = 22400 cm^3$$

$$N_{moli} = \frac{V}{V_{mol}} = 1.5 \ \ 10^{-6}$$

$$N_{atoms} = 1.5 \ \ 10^{-6} N_A = 9 \ \ 10^{17} \times 2$$

For comparison the total number of ions $N_{DT} \approx 2 \ 10^{21}$

The mass density of the gas is

$$n_{atoms} = \frac{9 \ \ 10^{17} \times 2}{0.03 cm^3} = 6 \ \ 10^{19} cm^{-3}$$

$$\rho = n_{atoms} m_i = 6 \ \ 10^{19} \times 2.5 \times 1.67 \ 10^{-24} g / cm^3 = 2.5 \ 10^{-4} g / cm^3$$

Correction of first test / 6th question

6) Assuming that at stagnation the radius of the hot spot is 20 μm.
What is the final pressure?

$$pV^{\gamma} = const \qquad p_{com} = po\left(\frac{V_o}{V_{com}}\right)^{\gamma}$$

$$V_o = 0.03\ cm^3 \qquad V_{com} = \frac{4}{3}\pi(20\,\mu m)^3 = 3.4\ 10^{-8} cm^3$$

$$p = 1Bar\left(\frac{0.03}{3.4\ 10^{-8} cm^3}\right)^{1.67} = 8.5\ \ 10^9 Bar = 8.5 GBar$$

$$\frac{\rho_{com}}{\rho_0} = \frac{V_0}{V_{com}} = 8\ 10^5$$

$$\rho_{com} = 8\ 10^5 \rho_0 = 200 g / cm^3$$

$$TV^{\gamma-1} = const$$

$$T = T_0\left(\frac{V_o}{V_{com}}\right)^{\gamma-1} = 0.025eV\left(\frac{0.03}{3.4\ 10^{-8} cm^3}\right)^{0.67} = 240eV$$

Correction of first test / 6th question

6) How much energy has been spent in compression of the fuel and in heating of the hot spot?

FINAL CONDITIONS

- $M \approx 0.41$ mg
- $\rho_{fin} / \rho_{in} \approx 1000$
- $\rho_{fin} \approx 250$ g/ cm^3
- $V_{fin} \approx 4/3\,\pi\,(R_{fin}^3 - R_o^3) \approx 1.6\ 10^{-6}$ cm^3
- $R_{hot\text{-}spot} \approx 10\ \mu$m
- $R_{fin} \approx 72\ \mu$m
- $\Delta r \approx 60\ \mu$m (A =1.2)
- $\rho_{fin} R_{fin} \approx 1.8$ g/ cm^2
- T_{fin}=240eV

$$\varepsilon_{compr} = 0.35\alpha M_f \rho_f^{2/3}\ MJ = 17kJ$$

$$\varepsilon_{chauf} = 110 M_h T_{keV}\ MJ = 10kJ$$